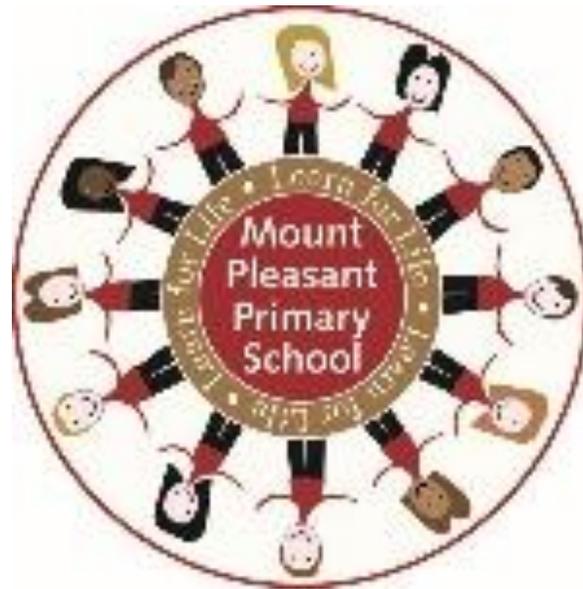


# Mount Pleasant

Teaching Multiplication Tables Guidance



## Introduction

The main aim of our whole-school multiplication tables approach is to teach multiplication tables in a way that supports both recall and application. Many children struggle to retain multiplication tables facts and can often feel overwhelmed by the sheer volume of facts to recall. However, when teachers in England are surveyed and asked to talk about effectively teaching times tables, conversation often ends up revolving around testing times tables, as though teaching and testing are the same thing. The difficulty with using testing as a means of improving recall is that, although it may motivate those who can already rote learn to increase their speed and confidence, it does not support those children who struggle with rote learning in making the connections needed to access tables facts. In fact, experience shows that it can often lead them to feel even more overwhelmed. It is to this end that we have put in place a system where multiplication tables and their underlying skills along with wider number relationships are taught and not just caught. Research into multiplication tables and memory has found the following:

- Maths facts are held in the working memory section of the brain (Beilock, 2011; Ramirez et al, 2013, cited in Boaler, 2015) until they are transferred to our long-term memory and to do this they need to understand the relationships between numbers and work flexibly with these, so they can transfer knowledge to long term memory.
- If such facts are not regularly brought out of our long-term store and back into our working memory, they can begin to be forgotten. The act of bringing facts back from our long-term memory to be applied in our working memory reinforces the strength of the long-term memory.
- Rote learning can encourage children to see multiplication tables facts as isolated pieces of information which, for some, prevents them from making use of the relationships within them. A typical example of this is the child who is unable to work out  $13 \times 7$  because they don't know their thirteen times table.

To ensure that our whole-school approach addresses these findings, we have put in place a strategy-based system for teaching multiplication tables and number relationships, which is consistent with our wider *White Rose Maths Curriculum*.

## What does research say are the obstacles to successful recall of multiplication tables?

**Over-reliance on skip counting:** many children, particularly those who struggle with recall, over-rely on skip counting and this makes it difficult to access multiplication table facts when not in order. It also means they often fail to recognise or make use of the commutative aspect of multiplication ( $2 \times 3 = 3 \times 2$ ) and instead use time-consuming, inefficient methods to access multiplication table facts.

**Inflexible multiplication strategies:** Key Stage Two children struggle to use flexible multiplication strategies because they do not have the required mental strategies in place from Key Stage One.

**Over-reliance on formal written methods:** children in Key Stage Two over-rely on formal written methods, instead of thinking logically about the numbers and concepts involved.

**Overload:** children feel overwhelmed by the number of multiplication tables facts to learn.

**Application:** children struggle to apply multiplication tables knowledge within related contexts such as division and fractions. This bringing of the knowledge back into working memory is crucial.

## How will we address these obstacles?

**Over-reliance on skip counting:** Give children multiple ways to access and apply tables facts by exploring different relationships early on, so skip counting becomes one aspect of multiplication rather than the primary strategy. When introducing concepts and strategies, use practical equipment and represent strategies visually to help children make connections and visualise relationships.

**Inflexible multiplication strategies:** directly teach mental arithmetic strategies and number relationships based around doubling, halving and number bonds.

**Over-reliance on formal written methods:** Explore a range of strategies from an early age; continue to develop mental methods which can be used for both arithmetic and reasoning questions throughout Key Stage Two.

**Overload:** have single table based system alongside main curriculum tables teaching.

**Application:** use a range of visual representations

## What are the key factors for success?

There are key skills upon which multiplication strategies rely. These must become automatic if strategies are to be used successfully to retrieve table facts at speed. They are:

- inverse relationships
- partitioning
- doubling/ halving
- number bonds within ten/multiples of ten
- bridging

A strong understanding of place value is also required in order for tables facts to be applied with larger numbers and decimals.

Research has shown that lack of confidence with these underlying skills significantly impaired a child's ability to develop alternative strategies for accessing times tables facts. Therefore, these key skills must be given a high priority throughout Key Stage One and Two.



## Multiplication Tables at Mount Pleasant

<b>Foundational Facts (Everyone Can)</b>	Monday to Thursday pupils to complete <i>Everyone Can</i> sheet for 10 minutes outside of maths lesson, which in the first term will be multiplication tables and their underlying skills: number bonds, double, halve, multiplication tables and division tables
<b>Counting</b>	Drawn from Mount Pleasant Counting Expectations (based on National Curriculum expectations for the year group) Could be link to main maths or Number Knowledge; could be stand alone Counting stick In sequence Random sequence Forwards backwards Teach the tables using a blank multiplication wheel; patterns; repeated addition
<b>Number Relationships</b>	From Monday to Thursday each year group will have ten minute session on number relationships and mental arithmetic including number bonds, doubling, halving and partitioning.
<b>Multiplication Tables Lesson</b>	Prior to Friday's multiplication tables test, children to be taught a direct skill for multiplication tables, for example $\times 9$ by $\times 10$ subtract $\times 1$ or to find $\times 8$ you can $\times 4$ then double.
<b>Multiplication Tables Test</b>	On a Friday children complete test, mark it and scores are recorded Children who pass the test move onto the next one; they get a certificate; and this is recorded on class tracker
<b>Homework</b>	On a Friday multiplication facts sheet and homework task are sent out (paper or dojo) for children to practice linked to their progress in tests Children start at $\times 2$
<b>TTRS</b>	Maths Lead sets TTRS questions based on the tables pupils are on from test results
<b>Maths Assembly</b>	Maths Lead to hold regular assemblies focusing on multiplication tables (at first) where we will have team colour quizzes and award a star TTRS performer (from own time not set class time) per class per week
<b>TTRS Competitions</b>	In-school and trust-wide TTRS competitions to continue to raise profile
<b>Training</b>	Maths Lead to lead training within first half term on the system and general good practice for teaching multiplication tables
<b>Monitoring</b>	Maths Lead to monitor test scores Monthly sound checks / MTC tests in Y3 and Y4

# Strategies for teaching underlying skills:

- inverse relationships
  - doubling
  - halving
- number bonds

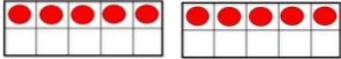


## Year 1 Guidance

### Underlying Skill: inverse relationships

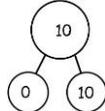
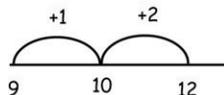
Notes	Concrete	Visual – moving towards Abstract
<p>Children should work practically to divide by 2, 5 and 10 and draw what they have done in ways that are meaningful to them.</p> <p>Teacher can model using part whole models/bar models, talking about the parts and whole amounts and linking it to multiplication through the discussion.</p> <p>Discuss the idea of equality and give examples where groups are unequal to discuss.</p>	<p><u>Sharing:</u> E.g. I have 8 stickers and I share them between my 2 friends. How many stickers do they get each? Share practically and then explore how it could be shown visually (drawing). Teacher can demonstrate how it would look in a part whole or bar model. Talk about what is the whole amount and what are the parts?</p> <div style="text-align: center;"> </div> <p><u>Grouping:</u> Problems involving grouping should also be included so children can use practical apparatus to act out the problem. The bus had room for 2 people on each seat. If there were 8 children getting on the bus, how many seats would they fill? Take off 2 children each time to fill a seat and teacher can draw part whole model as doing it. Discuss the whole amount and the parts.</p> <p>Relate it to everyday problems. Also, give examples solved incorrectly. E.g. Share some cubes out but give one person more than the others. Discuss if this is fair and relate to the idea of equal groups for both sharing and grouping.</p>	<p><u>Sharing:</u> Encourage children to draw the children (stick people or smiley faces) and crosses to represent the objects. Teacher can model recording on part whole or bar models. E.g. 8 shared between 2.</p> <p>When discussing sharing between 2, make the link to halving. Also, use bar models or part whole models to represent practical situations where sharing is between 5 or between 10.</p> <p><u>Grouping:</u> Encourage children to work through the problem and draw, for example, squares to represent the seats on the bus, then put cubes on them. Eventually it can be recorded with crosses to represent the cubes.</p> <div style="text-align: center;"> </div> <p>Teacher can model on part whole or bar models but children may use their own drawings to represent.</p> <div style="text-align: center;"> </div>

**Underlying Skill: doubling**

Notes	Concrete	Visual to support Abstract
<p>Ensure doubles are covered before multiplication.</p> <p>Expectation that children know double to 10+10.</p> <p>Lots of singing/ doubles raps/ fingers (for doubles within 10).</p> <p>Vocabulary- 'lots of' etc to be used with doubles.</p> <p>Bridging can also be used to develop mental doubling. (See page 18).</p>	<p>Use a range of concrete apparatus, e.g. Numicon, small world objects to show doubles. Show me double 3 with Numicon, with counters, with teddies etc.</p> <p>Use fingers. Show me 4. Now show me double 4. If <math>5+5=10</math>, what would <math>6+6</math> be? How do you know?</p>  <p>Show doubles in an array. Make one half of the array. Ask children to predict the whole amount and add the other half to check. Include and discuss examples of two numbers that aren't doubles (unequal groups). E.g. Which set shows double 5? How do you know?</p> <p>Make an even number but hide half under a pot/cover. Show half. Predict how many will there be altogether when remove pot/cover?</p> <p>Hide half of a snake made of beads or a tower of cubes behind your back.</p> <p>This is half my snake/tower.</p> <p>What does my whole snake/tower look like? (Can play as game in pairs).</p> <p>Dice can be used to double. E.g. Can children quickly say what the double is when shown one die? Can they recognise the doubles dominoes and match them to the total, calculations? etc.</p>	<p>Use dominoes to show a double and ask children to complete picture and answer.</p>  <p>Double 1 = ____</p> <p>Number sentences could be match to dominoes or dominoes drawn to match number sentences. Draw the domino for double 5. How many spots do you think will be on double 6? Explain your thinking.</p> <p>Show one half of an array.</p>  <p>Ask children to imagine then draw the other half and predict what the double will be. Write the addition to match.</p>



**Underlying Skill: bonds to 10 (preparation for bridging)**

Notes	Concrete	Visual to Support Abstract
<p>Reinforce bonds to 10 throughout the year. Predict the pattern when writing numbers that bond to ten as this involves mentally moving the numbers, so is useful to prepare for bridging. When children are secure with number bonds they can move onto bridging with practical equipment, again predicting what will happen when the numbers are moved over to make ten.</p> <p>Bonds can also be constantly practised through songs, rhymes, stories and actions.</p>	<p>Place a range of real objects on part whole and bar models, discussing which is the whole amount and which are the parts. Put 10 counters in the 'whole' of the part whole model and ask children to close their eyes while you split them into the two parts. Hide one part under a tub and ask the children to say how many are hiding. How do they know? Pour out ten double sided counters and find the number sentence to match it, explaining why they chose it. Discuss the different ways children may 'see' it, e.g. 3(red) +7(blue), 3(blue) +7(red), 7(red)+3(blue). Place all the counters the same way up and turn one over at a time. Record systematically, asking the children to predict the next in the sequence. Eg. Turn one over, what will the number sentence be now? (1+9=10). Turn another over, record the number sentence (2+8=10). What do they think the number sentence will be if they turn another counter over? And another? This visualisation requires children to visualise the movement of numbers, so prepares for bridging.</p> <p><u>Bridging</u></p> <p>Use tens frames alongside number tracks/ 100 squares with cubes placed on top to show how to use the ten as a 'bridge' to rest at instead of counting on in ones. E.g. For 9+3, one of the 3 can be imagined moving onto the tens frame so the calculation becomes 10+2 instead of 9+3. It can then be physically moved. Children can predict what will happen then find out.</p> <p>Begin by adding a single digit to 9, e.g. 9+3, 9+4, 9+5 etc. Gradually add to other single digit numbers once confident.</p>	<p>Fill in missing numbers on part whole models/bar models. Use them to work out the parts and the whole in problems.</p> <p>___ + 6 = 10 6 + ___ = 10 10 = 6 + ___</p> <p>Predict and write the next number sentence each time. E.g. What will happen if we move one across (from the right part to the part on the left)?</p> <p>0+10=10 1+9=10 2+8=10</p>  <p>Explain why one side increases by one each time and the other decreases by one (because one is moving over from one side to the other). Use cubes on bar models or part whole models to show this happening.</p> <p><u>Bridging</u></p> <p>Teacher can also demonstrate on a numbered number line.</p> 

**Underlying Skill:** Subtraction from 10/Multiple of 10

**Notes**

Practise counting across the hundredsquare.

**Concrete**

Show in a range of ways: full tens frames, sticks of ten, cubeshorizontally across a 100 square etc.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

10-2=8. What would 20-2 be?

I know 10-4=6. What else do I know?

What would 20-6 be? Prove it. What about 30-6?

**Visual to Support Abstract**

Write the multiples of ten on an empty 100 square. Use 10-2 to help find 20-2, 30-2 etc and write them in.

10-2=8,

20-2=18

30-2=28.

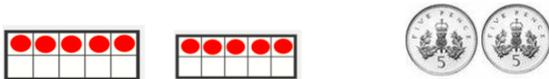
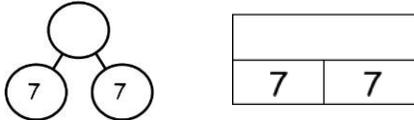
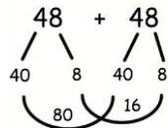
What do you notice?

What would be next in the pattern?

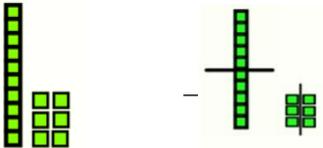
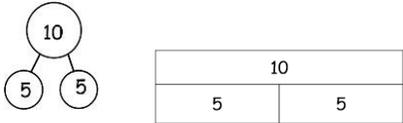
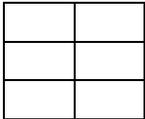
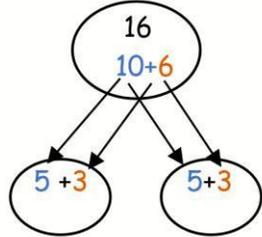
Use flashcards of full tens frames. Use the first to find 10-6. Predict what the number sentence will be when another ten is added and another etc.



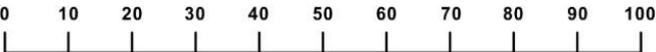
**Underlying Skill:** doubling

Notes	Concrete	Visual to Support Abstract						
<p>Revise doubles up to 5+5 and use with partitioning/place value to work out new facts up to 10+10. Extend and consolidate strategies throughout work with 2 digit numbers. Doubles may also be found by bridging (see page 29).</p> <p>As part of the work on doubles, teach children to visualise and exploit number relationships, by making use of near doubles: Show 6+6. What would 6+7 be?</p>	<p>If <math>5+5=10</math>, what would <math>6+6</math> be? How do you know?</p>  <p>What about <math>7+7</math>? (Draw attention to it as <math>5+5+2+2</math>, so known facts can be used). Represent with apparatus in part-whole models and bar models. Use a range of apparatus. Hide half for children to guess the whole amount. E.g. This is half my tower. What does my whole tower look like? Prove it.</p>  <p>Build in opportunities to consolidate doubling throughout addition work. E.g.</p> <table style="margin-left: 20px;"> <tr> <td>Addition of 3 single digits</td> <td><math>6+3+6</math></td> </tr> <tr> <td>2 digit + 1 digit</td> <td><math>36+6</math></td> </tr> <tr> <td>2 digit + 2 digit</td> <td><math>48+48</math></td> </tr> </table> <p style="text-align: right; margin-right: 100px;"><math>=80+16</math></p>	Addition of 3 single digits	$6+3+6$	2 digit + 1 digit	$36+6$	2 digit + 2 digit	$48+48$	<p style="text-align: center;">OR</p>  <p>Show a picture of half of an amount. Draw the rest of the objects/tower/number line etc. Put into bar models and part-whole models. Show on number line.</p>  <p>What is double 48? How did you work it out?</p>  <p>Did anyone find it a different way? (E.g. <math>48+40=88</math>. <math>88+8</math>).</p>
Addition of 3 single digits	$6+3+6$							
2 digit + 1 digit	$36+6$							
2 digit + 2 digit	$48+48$							
<p>Reasoning (to address misconceptions/mistakes or make predictions and generalisations).</p>	<ul style="list-style-type: none"> <li>• Erin says <math>46+46=86</math>. What is her mistake?</li> <li>• True or false: double <math>37 &gt; 60</math>. How do you know?</li> <li>• Double <math>7 &lt; \underline{\quad}</math>. What could it be? What couldn't it be?</li> <li>• Tianna says double 27 is 56 but Rory says he knows this can't be correct. Explain how he knows. (Double <math>6=12</math> so there should be 2 in the ones).</li> </ul>							

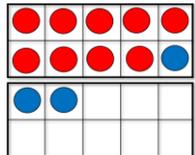
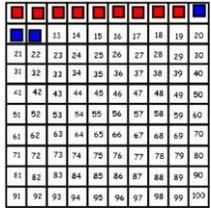
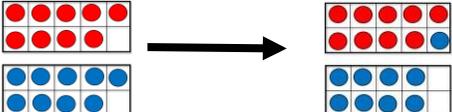
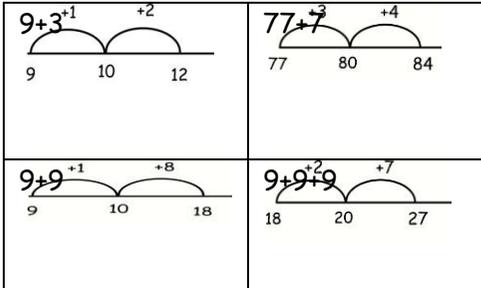
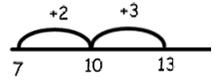
**Underlying Skill: halving**

Notes	Concrete	Visual to Support Abstract
<p>Use equipment where the ten group is obvious, such as tens frame, Numicon or cubes in sticks of ten with ones. Halving will also be carried out with shapes and other objects, but for the purpose of times tables, this programme focuses on number only. It is important to move beyond sharing out apparatus and use the structure of numbers to build mental strategies.</p>	<p>Use a range of concrete apparatus to halve initially by sharing on laminated part-whole and bar models. What is the whole amount? How many parts? What is important about the parts? Give cubes in two different colours. Build a tower. What would half your tower look like? Practise building, snapping in two to make two equal parts and comparing each part. Recap halves of numbers up to 10. Build me a tower that is half white etc. How can you check that half is white?</p> <p style="text-align: center;">Beau says these towers are half yellow? Is he right?</p>  <p>Repeat the above activities with numbers to 20. Use knowledge of halves to ten and place value to halve teens numbers up to 20. What is half of 10? So what would half of 12 be? (Halve the 10, halve the 2). What about 14? 16?</p>  <p>What about half of 26, 84 etc.</p>	 <p>Colour half of a tower picture or grid. Write the fractions sentence to go with it.</p>  <p>Half of 16</p> 
<p>Reason to address misconceptions or help make generalisations.</p>	<ul style="list-style-type: none"> <li>David has 26 cubes. He says if he gives half away, he will have 16 left. Explain his mistake. Prove it.</li> <li>Jess has three towers of bricks. One is 14 cubes tall, one is 15 cubes tall and one is 17 cubes tall. Half of one tower is white. Which tower? Explain how you know.</li> </ul>	

**Underlying Skill: Subtraction from a Multiple of Ten**

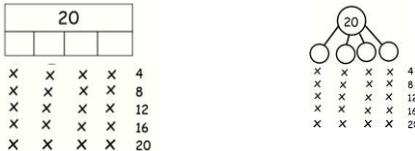
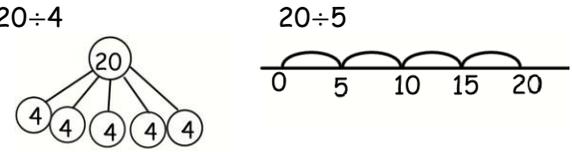
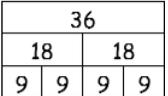
Notes	Concrete	Visual to Support Abstract																																																																																																															
<p>This skill links to the later strategy for finding the ninth multiple, so it must become automatic. It also consolidates number bonds to ten.</p>	<p>Show this in a range of ways: full tens frames, number pieces, sticks of ten cubes horizontally across 100 square etc. E.g. <math>10-2=8</math>. What would <math>20-2=</math></p> <table border="1" data-bbox="497 368 779 628"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table>  <p>Give an empty 100 square. Write on the multiples of 10. If <math>10-2=8</math>, write in <math>20-2, 30-2</math> etc.</p> <table border="1" data-bbox="524 743 1122 794"> <tr><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr> </table> <p>Look at the number line. If I jump back 2 from 10, where will I land? What about 20,30 etc?</p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	0	10	20	30	40	50	60	70	80	90	100	<p>Write the multiples of ten on an empty 100 square. Use it work out <math>10-2, 20-2, 30-2</math> etc.</p> <p><math>40-6=</math></p> <p>10 10 10 10 <math>-6= 4</math></p> <p>Draw place value counters to work out, gradually moving towards mental methods. Show where <math>30-2, 40-2, 50-2</math> etc will be on an empty number line.</p> 
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<p>Reason to address misconceptions or generalise.</p>	<p>I use my empty 100 square/number line to work out that <math>50-4=56</math>. What mistake have I made? Use concrete equipment to prove it.</p> <p>Look at the pattern: <math>10-3=7</math>  <math>20-3=17</math>  <math>30-3=27</math></p> <p>What would come next? Prove it.</p> <p>I know <math>10-6=4</math>, what other subtraction facts can I work out? Can these help me find any addition facts?</p>																																																																																																																

**Underlying Skill: Bridging**

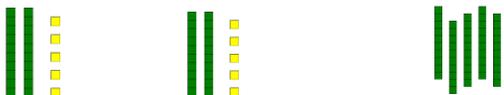
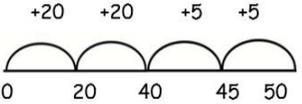
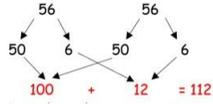
Notes	Concrete	Visual To Support Abstract
<p>This reinforces the key skills of number bonds, partitioning and place value, whilst allowing children to manipulate numbers and develop mental methods.</p> <p>Before learning to bridge with 2 digit numbers, children must be able to identify the ten before and after it. They must also be able to add single digits to multiples of ten and know how many would be needed to reach the next ten.</p>	<p>Use tens frames alongside number lines and 100 squares with cubes placed on top to show how to use the ten as a 'bridge' to rest at.</p>  <p>Begin by adding a single digit to 9. Show it on 2 tens frames or with 9 counters on a number track to 10 and 3 other loose counters. Ask the children to predict what the total will be when put together, so they can visualise the movement of the counters. Then physically move it, so 9+3 becomes 10+2. Repeat with 9+4, 9+5 etc.</p> <p>Gradually introduce other numbers (8+3, 8+4 etc) once confident.</p> <p>It can be also explored through placing different coloured cubes on top of tens frames or 100 squares.</p>  <p>The strategy can then be used with larger numbers, e.g. 18+4, 27+5, 79+7 when adding single digits to 2 digit numbers.</p>  <p>Bridging can also be used to developmental doubling. What would 9+9 be? Explain how you know.</p> <p>Once established, build in opportunities to triple numbers by doubling then bridging the last number, e.g. 8+8+8 as this lays strong foundations for the three times table, as well as consolidating doubles.</p> <p>Bridging can be taught as part of the work on addition of single digits to 2 digits numbers.</p>	<p>Colour squares on a 100 square to show cubes added or draw counters added in a tens frame. An arrow can be used to show how one of the 3 has moved to the 9 to make an equivalent (easier) calculation - 10+2.</p> $\overset{1}{\curvearrowright} 9 + 3 =$ <p><math>10 + 2</math></p> <p>It is also useful to show it on a number line, as children can draw their own as a stage between the concrete and the abstract to help them imagine the process.</p> 
<p>Reason to address misconceptions or to generalise.</p>	<p align="center">David worked out 7+5 like this. Is he right? Explain your thinking.</p> 	

## Year 3 Guidance

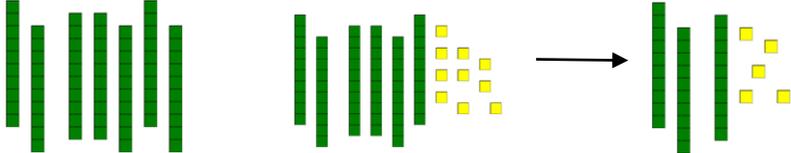
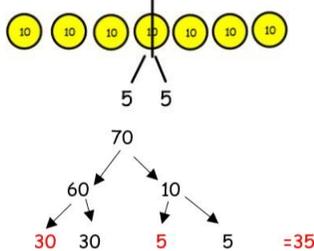
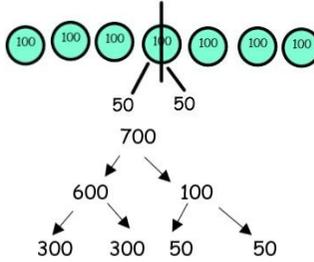
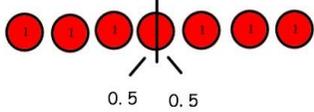
**Underlying Skill:** inverse relationships

Notes	Concrete	Visual	Abstract
<p>Use the same strategies as in Y2 to explore the sharing/ grouping link.</p> <p>Include division by 1 in both grouping and sharing contexts. These strategies can also be used for work on fractions. When working with larger numbers, e.g. 200, also talk about it as 20 tens (200) when sharing so children see the link to dividing 20 ones.</p>	<p><u>Sharing:</u> When sharing practical apparatus as part of real life problems it can be useful to share in an array on a part whole model or bar model. Count in multiples down the side or use them to count whilst sharing. Gradually move towards using multiples to solve, without concrete or visuals.</p> <p><u>Grouping:</u> Divide by grouping practically by taking off groups of a given multiple using a range of representations to show the repeated subtraction.</p> <p>E.g. How many groups of 4 in 20? This can be solved practically using a part whole model, bar model or a number line.</p> <p>Use place value counters/Dienes to make links to known facts when solving sharing and grouping calculations with larger numbers.</p> <p style="text-align: right;">E.g. Use bar/part whole model to solve <math>200 \div 4</math> or 20 tens <math>\div 4</math></p> <p>Use to explore problems, including those with remainders, and write calculations for missing numbers e.g. <math>\_ \div 4 = 50</math>.</p> <p>Find <math>100 \div 10</math>, <math>200 \div 10</math> <math>300 \div 10</math>. What do you notice?</p> <p>Links between tables should be explored when dividing as well as multiplying. E.g. Explore practically and visually the link between dividing by 3 and dividing by 6 and link this to work on fractions.</p>	<p><u>Sharing:</u></p> <p><math>20 \div 4</math> or <math>\frac{1}{4}</math> of 20</p>  <p><u>Grouping:</u></p> <p>Grouping can also be shown on a number line.</p> <p><math>20 \div 4</math></p>  <p>Talk about 20 tens, 20 hundreds, when working with larger numbers to support the link to ones. E.g. <math>2000 \div 4</math> is <math>2000</math> (20 hundreds) <math>\div 4</math> etc.</p> <p>E.g. Use visuals to show:</p> <p><math>12 \div 3</math>   <math>12 \div 6</math>.  <math>18 \div 3</math>   <math>18 \div 6</math>.  <math>24 \div 3</math>   <math>24 \div 6</math>.</p> <p>What do you notice? Show it on a bar model. Explore other numbers. What do you find out?</p> <p>E.g. Use concrete apparatus to show <math>24 \div 4</math>. How can this help you find <math>24 \div 8</math>? (Also explore the doubling/halving relationships with fractions).</p>	<p>Practical and visual representations can be used to solve missing number problems with division until the relationships are fully understood and these can be solved in the abstract. E.g. <math>\_ \div 4 = 5</math>. <math>30 \div \_ = 5</math></p> <p>What is the whole amount and what are the parts? Now draw the part whole model/bar model to find the missing number. Strategies for <math>\div 4</math>/finding a quarter (halving and halving again) or <math>\div 8</math> (halving, halving and halving again) should be explored practically on bar models, before recording visually.</p> <p>E.g. <math>36 \div 4</math> or <math>\frac{1}{4}</math> of 36.</p>  <p>Word problems can be written to match the visual or calculation and vice versa.</p>

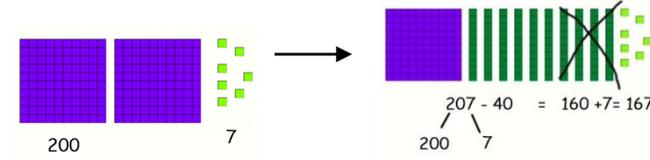
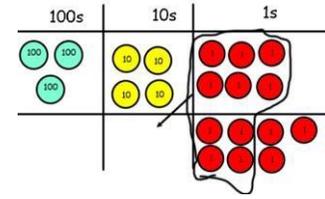
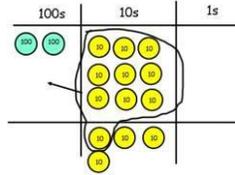
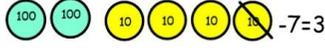
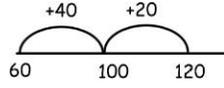
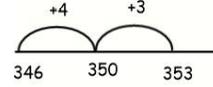
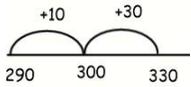
**Underlying Skill: doubling**

Notes	Concrete	Visual to Support Abstract
<p>Revise doubles up to 12 +12 using strategies from Year 2 if necessary (p26). Also include near doubles.</p> <p>Double multiples of 10, using what is known about single digits.</p> <p>Double multiples that end in 5, (25, 45 etc)</p> <p>Build in opportunities to double/near double when working with place value, addition, subtraction money and measures.</p> <p>Also, include when using formal written methods, so children continue to use mental skills.</p> <p>When doubling, link to <math>\times 2</math>, so children develop mental methods for <math>\times 2</math>.</p>	<p>If <math>20+20 = 40</math>, then what is <math>25 + 25</math>? Use Dienes or place value counters to prove it.</p>  <p>What does this show? What if each of the counters was worth 10/was replaced with a tens counter? What would <math>60+60</math> be? How do you know? These represent half my sweets. How many sweets do I have in total? Predict, then prove it.</p> <p>Double 64 = <math>100+20+8</math>.</p>  <p>What number facts could help me find double 64? (Link double 6 to double 60 using practical equipment to begin with).</p> 	 <p>What is double 56? How did you work it out? Did anyone find it a different way?</p>  <p>Place value counters may be drawn alongside to illustrate for a short period, if relationships are not fully understood.</p> <p>If double <math>56=112</math>, what would <math>56+57</math> be?</p> <p style="text-align: right;">So, what would <math>64 \times 2 = ?</math></p> <p>Revise doubling throughout all tables by exploring and making decisions about different ways to solve calculations, e.g. <math>4 \times 9 =</math></p>
<p>Reason (to address misconceptions or to make predictions/generalisations).</p>	<p>True or false: double 87 &gt; 140. How do you know?</p> <ul style="list-style-type: none"> <li>• Double <math>12 &lt; \underline{\quad}</math>. What could it be? What couldn't it be?</li> <li>• Sally says <math>46 + 46 = 82</math> but Rory says he knows this can't be correct. How does he know?</li> <li>• Asif has twenty six 2p coins. He thinks he can find out how much he has by doubling. Is he right?</li> </ul>	

**Underlying Skill: halving**

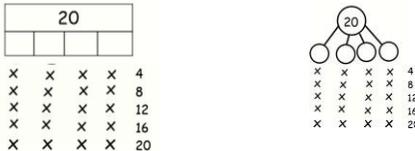
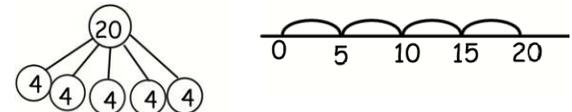
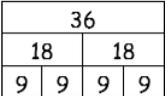
Notes	Concrete	Visual to Support Abstract
<p>Use equipment such as Dienes or place value counters to link 2-digit numbers to known facts.</p> <p>If children are not secure from Year 2, build in opportunities to halve 2-digit numbers by halving the tens and halving the ones.</p> <p>Ensure two-digit numbers with an odd number of tens are included.</p> <p>Doubles and halves relationships can be revised when exploring equivalent fractions.</p>	<p>Use a range of concrete apparatus to solve problems and halve - initially by sharing. What is the whole amount? How many parts? What is important about the parts? Write word problems that different representations/ calculations could represent.</p> <p align="center">Share place value counters in an array (see pages 11-14) to show the link between dividing by 2 and halving.</p> <p>E.g. Half of 70. Use Dienes to show exchange in the final 10.</p>  <p>Also use place value counters and predict what will happen to the last ten. Halve 2 digit numbers where the number of tens is odd, e.g. 36, 54, 78 (e.g. Halve the 30, halve the 6 etc).</p> <p>Use a counting stick, Cuisenaire rods etc to find out how many halves are in 2? 3? 4? etc. What do you notice? Why does this happen?</p>  <p>Use decimal place value counters when exploring tenths. Find half of one. How can we write it as a decimal?</p> <p>Halve odd numbers, showing 0.5, e.g. half of 3 = 1.5 or 1 1/2.</p>	<p>E.g. Half of 70. Draw Dienes or place value counters.</p>  <p>What would half of 700 be?</p>  <p>Half of 7?</p>  <p>Write down all the fractions equivalent to half. What do you notice?</p>
<p>Reason to address misconceptions or help make generalisations.</p>	<ul style="list-style-type: none"> <li>• David has 96 cubes. He says if he gives half away, he will have 43 left. Is he right? Explain his mistake. Prove it.</li> <li>• Jess has three towers of bricks. One is 14 cubes tall, one is 15 cubes tall and one is 17 cubes tall. Half of one tower is 7.5. Which tower? Explain how you know.</li> </ul>	

**Underlying Skills: Subtraction from Multiples of 10 and 100 and Bridging**

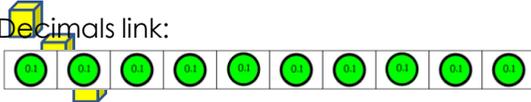
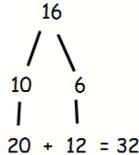
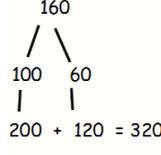
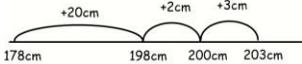
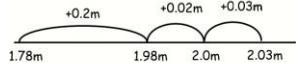
Notes	Concrete	Visual to Support Abstract
<p>Subtraction from multiples of ten is required to be automatic for x9 by subtracting to find the ninth multiple and for recognising relationships when estimating, multiplying and dividing with larger numbers. (E.g. knowing that there will be <math>24 \times 4 = 96</math> because <math>25 \times 4 = 100</math>). Bridging is required for x3 by tripling.</p>	<p>Discuss strategies during formal written methods, so children learn to decide when to work mentally and when to use a written method. Use Dienes or place value counters to make the link to known facts.</p> <p><b>3-digit - 1 digit - E.g. 240-7 or 300-4.</b></p>  <p><b>E.g. 3 digits - tens - 207-40.</b></p>  <p><b>Bridging</b></p> <p><b>3-digit + 1 digit - 346+7</b></p>  <p><b>3 digit + tens - 290+40.</b></p> 	<p>Draw the Dienes or place value counters to make link to known facts. E.g. <math>240-7=233</math> because <math>10-7=3</math>.</p>  <p>Also, show on number lines. <u>Bridging</u></p> <p>Links can be made to single digit facts when working with multiples of ten.</p>  <p><b>3-digit + 1 digit - 346+7</b></p>  <p><b>3 digit + tens - 290+40.</b></p>  <p>Doubles can also be consolidated during this work.</p>
<p>Reasoning - address misconceptions or make generalisations.</p>	<p>Link to known facts, I know <math>40-7=3</math> so <math>240-7=?</math> What would <math>340-7</math> be?  <math>300-4=296</math>, <math>400-4=396</math>, <math>500-4=496</math>. What do you notice? What would the next calculation be?                  Spot the mistake:</p>	

## Year 4 Guidance

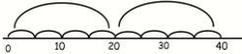
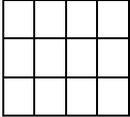
**Underlying Skill:** inverse relationships

Notes	Concrete	Visual	Abstract
<p>Use the same strategies as in Y2 to explore the sharing/grouping link.</p> <p>Include division by 1 in both grouping and sharing contexts. These strategies can also be used for work on fractions. When working with larger numbers, e.g. 200, also talk about it as 20 tens (200) when sharing so children see the link to dividing 20 ones.</p>	<p><u>Sharing:</u> When sharing practical apparatus as part of real life problems it can be useful to share in an array on a part whole model or bar model. Count in multiples down the side or use them to count whilst sharing. Gradually move towards using multiples to solve, without concrete or visuals.</p> <p><u>Grouping:</u> Divide by grouping practically by taking off groups of a given multiple using a range of representations to show the repeated subtraction.</p> <p>E.g. How many groups of 4 in 20? This can be solved practically using a part whole model, bar model or a number line.</p> <p>Use place value counters/Dienes to make links to known facts when solving sharing and grouping calculations with larger numbers.</p> <p style="text-align: right;">E.g. Use bar/part whole model to solve <math>200 \div 4</math> or 20 tens <math>\div 4</math></p> <p>Use to explore problems, including those with remainders, and write calculations for missing numbers e.g. <math>\_ \div 4 = 50</math>.</p> <p>Find <math>100 \div 10</math>, <math>200 \div 10</math> <math>300 \div 10</math>. What do you notice?</p> <p>Links between tables should be explored when dividing as well as multiplying. E.g. Explore practically and visually the link between dividing by 3 and dividing by 6 and link this to work on fractions.</p>	<p><u>Sharing:</u></p> <p><math>20 \div 4</math> or <math>\frac{1}{4}</math> of 20</p>  <p><u>Grouping:</u></p> <p>Grouping can also be shown on a number line.</p> <p><math>20 \div 4</math></p>  <p>Talk about 20 tens, 20 hundreds, when working with larger numbers to support the link to ones. E.g. <math>2000 \div 4</math> is <math>2000</math> (20 hundreds) <math>\div 4</math> etc.</p> <p>E.g. Use visuals to show:</p> <p><math>12 \div 3</math>    <math>12 \div 6</math>  <math>18 \div 3</math>    <math>18 \div 6</math>  <math>24 \div 3</math>    <math>24 \div 6</math>.</p> <p>What do you notice? Show it on a bar model. Explore other numbers. What do you find out?</p> <p>E.g. Use concrete apparatus to show <math>24 \div 4</math>. How can this help you find <math>24 \div 8</math>? (Also explore the doubling/halving relationships with fractions).</p>	<p>Practical and visual representations can be used to solve missing number problems with division until the relationships are fully understood and these can be solved in the abstract.</p> <p>E.g. <math>\_ \div 4 = 5</math>. <math>30 \div \_ = 5</math></p> <p>What is the whole amount and what are the parts? Now draw the part whole model/bar model to find the missing number. Strategies for <math>\div 4</math>/finding a quarter (halving and halving again) or <math>\div 8</math> (halving, halving and halving again) should be explored practically on bar models, before recording visually.</p> <p>E.g. <math>36 \div 4</math> or <math>\frac{1}{4}</math> of 36.</p>  <p>Word problems can be written to match the visual or calculation and vice versa.</p>

**Underlying Skills:** Doubles and Halves, Bridging, Subtraction from a Multiple of 10,100,1000

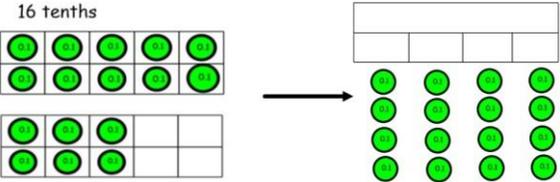
Notes	Concrete	Visual to Support Abstract
<p>Build in opportunities to double, halve and bridge single digit numbers in formal written methods. Revise these skills with partitioning and place value in mental calculations with larger whole numbers and decimals in the context of money, measures and time.</p> <p>Revise and consolidate the importance of doubling and halving within the tables (i.e. between 5 and 10, 2 and 4, 4 and 8, 3 and 6) during work on fractions. E.g. Draw bar models/fraction walls linking fifths and tenths, thirds and sixths etc. Investigate whether finding one tenth can help to find one fifth or one third can help you to find a sixth. How could this help with division facts?</p>	<p>Use place value counters, Dienes, Numicon etc to represent and solve problems. Include doubling, halving, bridging during work with money using coins practically. (See Y2, page 30, if practical work needed on bridging). E.g. doubling. <math>2 \times 16 = 2 \times 10, 2 \times 6</math> so double 160 = double 100, double 60.</p> <p><u>Fractions link:</u> Use Cuisenaire rods/cubes to revise doubling and halving with equivalent fractions. Make all the fractions you can that show one half. On a counting stick or number line marked in ones, show 5. How many halves will there be? What do you notice? Use cuisenaire rods (or cubes) to represent tenths in a whole amount. How many fifths will there be? E.g. This is <math>\frac{3}{10}</math>. Make the whole and put it in a bar model. How could you find out how many fifths is it equivalent to?</p> <p><u>Decimals link:</u></p>  <p>Share out between 2 people. They get <math>\frac{5}{10}</math> each which is the same as half. How will you write half as decimal? So how would one fifth be represented as a decimal? Show it on a bar model.</p>	<p>Draw place value counters or Dienes to help solve initially, moving towards a mental method.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="1352 347 1491 539"> <p>Double 16</p>  </div> <div data-bbox="1648 347 1809 539"> <p>Double 160</p>  </div> </div> <p>E.g. Kemi's room is only 1.78 m wide, but Kyra's is 25cm longer. How long is Kyra's room?</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="1319 639 1621 703">  </div> <div data-bbox="1648 632 1944 695">  </div> </div> <p>203cm = 2.03m.</p> <p><u>Fractions links:</u> Write down all the fractions equivalent to one half. What do you notice? How many halves are in 2? 3? 4? etc. What do you notice? Why does this happen? Also, use doubles and halves links to explore other equivalent fractions. How many tenths are in one fifth? two fifths etc? How many sixths are in two thirds etc? Draw bar models/fraction walls to prove it.</p>

**Underlying Skill: the associative property**

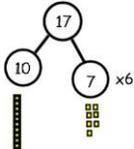
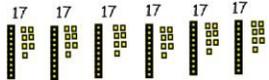
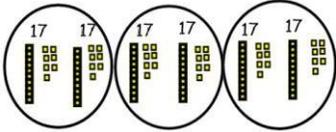
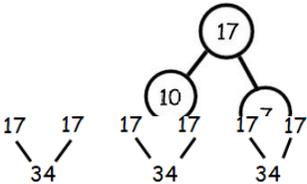
Notes	Concrete	Visual to Support Abstract
<p>Investigate and talk about the associative property (i.e. <math>8 \times 6 = 2 \times 4 \times 6</math> etc) practically. Use all known tables, beginning with those the children are likely to know well and moving on to new tables.</p>	<p>Make an array to show <math>10 \times 4</math>. True or false: it also shows 2 lots of <math>2 \times 20</math> or 2 lots of <math>4 \times 5</math>?</p> <p>Explore ways of breaking down arrays into smaller groups, beginning with familiar tables facts, then building onto new tables taught.</p> <p>Use multi-link to make cuboids to explore the associative property. E.g. make a cuboid to show <math>6 \times 2 \times 3</math>. Build one face, then predict how many cubes you will need. Make it to find out. Can you make any other cuboids with the same number of cubes? Write the calculation that they show.</p> <p>Ravi is going to build a cuboid with sides measuring <math>4 \times 5 \times 3</math>, whilst Ralph will build one with sides of <math>5 \times 5 \times 3</math>. Who will need the most cubes to build it? How do you know? Use cubes to show how <math>2 \times 30 = 20 \times 3</math>. (<math>2 \times 10 \times 3</math>). Extension: My cuboid is made of 40 cubes. How long could the sides be? Use cubes to find out. Could your multiplication and division facts help you work it out? Could one of the sides be 3 cubes long? How do you know? Show different cuboids (give dimensions) and ask the children to quickly work out on whiteboards its volume (by multiplying in the most efficient way). Discuss the different ways it was calculated.</p>	<p>Teacher could discuss ideas about what this might look like on a number line and bar model and model ideas about grouping. E.g. 40 is broken down into 4 lots of <math>2 \times 5</math> (<math>4 \times 2 \times 5</math>).</p> <p>Or 2 lots of <math>4 \times 5</math> (<math>2 \times 4 \times 5</math>).</p> <p>Draw circles round arrays to match the number lines. Match visuals to calculations and vice versa.</p>  <p>In pairs, read out calculations from cards for partner to draw (e.g. 2 lots of <math>3 \times 4</math> or <math>2 \times 3 \times 4</math>). Match the calculations to the visuals drawn together and explain how you made your choice.</p> <p>Build cubes to represent different calculations. E.g. Build a cuboid to show this: <math>3 \times 2 \times 4</math>. This is one face of my cuboid. Its total volume is less than 60 cubes. What length could its other side be? What could it definitely not be?</p> 

## Year 5 & 6 Guidance

**Underlying Skill:** inverse relationships

Notes	Concrete	Visual	Abstract
<p>Explore the relationships between largernumbers and decimals to show that, although the place value haschanged, the relationships stay the same.</p> <p>The doubles and halves relationships within the tables should be exploited tohelp with division.</p> <p>Include division by 1 and division by10 regularly.</p>	<p>Use strips or tens frames with 10 tenths shown (place value counters/Dienes tens) to represent</p> <p>1. Use these to show the effect of dividing by 10 resulting in decimal numbers. E.g. <math>4 \div 10</math> is 40 tenths <math>\div 10</math> which is 4 tenths. Ensure this is explored alongside the place value slider or grid and encourage children to visualise the number oftenths in whole numbers before calculating in theabstract.</p> <p>E.g. <math>1.6 \div 4</math>     1.6 or 16 tenths</p> <p>Sharing:</p>  <p>Use Dienes or larger place value counters to showthe same relationships with larger numbers. E.g. Use plave value slider or grid alongside placevalue counters to show the effect of dividing by1000, or to link known facts to work with larger numbers.</p> <p>Investigate factor relationships. E.g. True or false: <math>70 \div 20</math> can be found by dividing by 2 then dividing by 10. Prove it. Do youthink you could divide by 10 then divide by 2? Explain.</p>	<p>Use the visual as a step between theconcrete and the abstract and to explore relationships. E.g. <math>1.6 \div 4</math> or <math>\frac{1}{4}</math>of 1.6 could be visualised on a bar model, thinking of 16 tenths.</p>  <p>Select 3 and 4 digit multiples of ten/hundred as a starting point toinvestigate patterns in division.</p> <p>E.g.     Predict the next three <math>160 \div 4 =</math>     calculations. Explore how <math>161 \div 4 =</math>. partitioning into <math>160 + 1</math>, <math>162 \div 4 =</math>     <math>160 + 2</math> etc can help with     mental division.</p> <p>Factors should also be used to divide,focusing particularly on the relationship between <math>\div 10</math> and <math>\div 5</math> or <math>\div 20</math> and exploiting doubles and halvesrelationships within the tables. E.g. <math>\div 20</math> by <math>\div 10</math> then <math>\div 2</math>. <math>\div 5</math> by <math>\div 10</math> then doubling.</p> <p>Relationships can be shown visually. E.g. <math>80 \div 5</math></p>	<p>Practical and visual representations canbe used initially to solve missing number problems with division, so children can visualise the relationships when working in the abstract.</p> <p>E.g. <math>\_\_\_ \div 4 = 0.5</math>. What is the whole amount and what arethe parts?/How many are in each part?</p> <p>Look for relationships between numbers to make calculations easier tosolve. . E.g. <math>70 \div \_\_\_ = 3.5</math>. <math>70 \div 2 = 35</math>. <math>35 \div 10 = 3.5</math>.</p> <p>The bar model can show <math>35 \times 2 = 70</math>, so how many 3.5 will be in each 35? Use place value counters strips initially andcheck with a place value slider/grid. What do you notice? <math>50 \div 10 =</math> <math>60 \div 12 =</math> <math>70 \div 14 =</math> Why does this happen?</p>

**Linked Skill:** using Tables Facts with Larger Numbers and Decimals

Linked Objectives	Notes	Concrete	Visual to Support Abstract
<p>Y5 and 6 Multiply and divide numbers mentally drawing upon known facts also required for use in long and short multiplication and division. Use knowledge of factors to multiply.</p>	<p>Use the properties of multiplication and known facts to work with larger numbers and decimals in same way as in previous years.</p>	<p>E.g. Explore different ways of finding <math>17 \times 6</math>. E.g. <math>(10 \times 6) + (7 \times 6)</math>. <math>10 \times 6 = 60</math> <math>7 \times 6 = 42</math></p>  <p>Consolidate understanding of how factors can be used to multiply, e.g. <math>17 \times 6 = 17 \times 2 \times 3</math> (or <math>34 \times 3</math>). What would it look like drawn as <math>17 \times 3 \times 2</math>?</p>   <p>What would <math>170 \times 6</math> be? What about <math>1.7 \times 6</math>?</p>	<p><math>(10 \times 6) + (7 \times 6) = 60 + 42 = 102</math></p>  <p><math>30 \times 3 = 90</math> <math>4 \times 3 = 12</math></p> <p>Visuals such as these can be used to gain understanding of the strategy of doubling one side and halving the other to gain an equivalent calculation. E.g. <math>17 \times 6 = 34 \times 3</math>.</p>



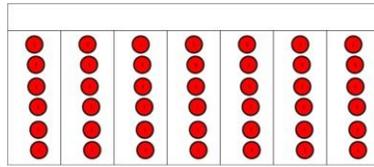
<p>Count forwards or backwards in steps of powers of 10. (Y5)</p> <p>Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 (Y5).</p>	<p>Link larger numbers and decimals to known facts. Use to consolidate previous table knowledge and relationships. E.g. look at 3 on counting stick and tripling link.</p>	<p>Use the counting stick and post-it notes to put on the 1<sup>st</sup>, 10<sup>th</sup> and 5<sup>th</sup> multiples as markers. See section 'Using the Counting Stick', page 7).</p> <p>Explicitly link tables facts to new facts through place value relationships. Place value counters and sliders can be used alongside this.</p> <p>E.g. 0.4 0.8 1.2</p>  <p>40 80 120</p> <p>This can also be used to explore the associative law. <math>7 \times 40 = (7 \times 4) \times 10 = 280</math>.</p> <p>Make links to known facts practically.</p>	<p>Draw a number line to represent the counting stick. Fill in missing numbers based on knowledge of relationships and place value, revising previous strategies and applying to larger numbers and decimals.</p> 
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Generate and describe linear sequences. (Y6)  
 Multiply and divide numbers by 10, 100 and 1000 (up to 3dp). (Y6).

Multiply one-digit numbers with up to two decimal places by whole numbers. (Y6).

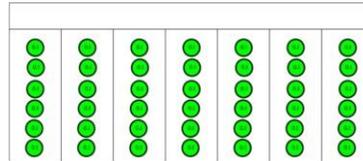
When working with decimals, Dienes equipment can also be useful, with a hundred block representing one whole, tens representing tenths and ones representing hundredths.

$7 \times 6 = 42$



$7 \times 0.6 = 4.2$

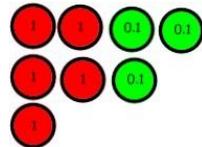
The array formation can also show  $6 \times 0.7$ .



If  $7 \times 0.6 = 4.2$ , what would  $7 \times 1.6$  be?  
 If  $7 \times 0.6 = 4.2$ , what would  $14 \times 0.6 =$  etc?

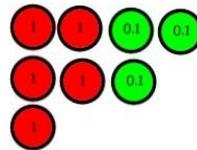
Use a place value grid multiplying and dividing by 10, 100 etc alongside practical division apparatus to support understanding of place value.

Once place value knowledge is secure, apply doubling and know strategies from halvingus tables. previous  $5.3 \times 15$



$$\begin{array}{r} 5.3 \times 15 \\ \phantom{5.3} \swarrow \searrow \\ 10 \quad 5 \end{array}$$

$5.3 \times 10 = 53$   
 $5.3 \times 5 = 26.5$   
 $53 + 26.5 = 79.5$



$$\begin{array}{r} 5.3 \times 15 \\ \phantom{5.3} \swarrow \searrow \\ 5 \quad 0.3 \end{array}$$

$5 \times 15 = 75$   
 $0.3 \times 15 = 4.5$   
 $75 + 4.5 = 79.5$

Draw place value counters alongside if needed to make links between known facts and decimals.

If  $7 \times 6 = 42$ , what would  $7 \times 60$  be? What about  $7 \times 0.6$ ?  $7 \times 6000$ ?  
 Predict what will happen on place value slider or place value grid.

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	2	8	0

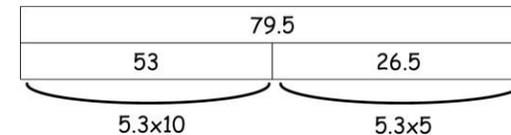
Draw arrays /rectangles where each square is worth 0.1/10,100,1000 etc to prove it.

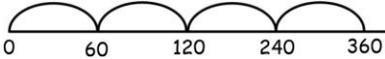
Write fact families that could be represented by the arrays (include division facts).

Use arrays, bar models and number lines to solve missing number problems, e.g.  $\_\_\_ \div 0.7 = 6$ .

Draw place value counters alongside calculations, moving onto bar models to reveal relationships, before working in the abstract.

E.g.  $5.3 \times 15$ .

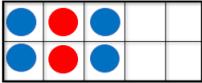
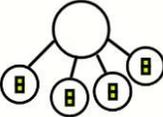
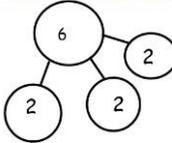
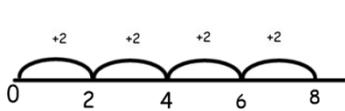


<p>Convert between different units of metric measure, including problems involving conversion of time. (Y5) Solve problems involving calculation and conversion of units of measure (length, mass, volume, time), using decimal notation (up to 3dp). (Y6).</p>	<p>Look for contexts which may lend themselves to consolidation of particular tables.</p>	<p>E.g. Use work with conversion of time to revise x6 table. E.g. Time: 6 12 18 24 etc  60 120 180 240</p>	<p></p> <p>Use a number line and knowledge of 6x table to convert between units of time, e.g. Convert 265 minutes into hours.</p>
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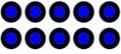
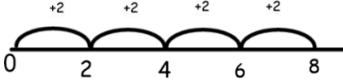
# Strategies for teaching specific multiplication Tables



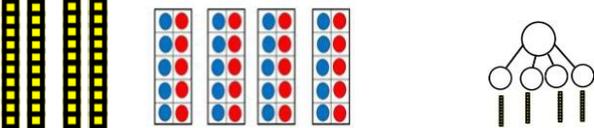
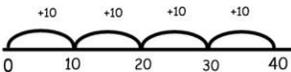
## Counting in Twos- Relationship Between Repeated Addition and Skip Counting.

Notes	Concrete	Visual to Support Abstract																								
<p>The Year One curriculum does not state that children must be able to use the multiplication sign, but instead focuses on the concept of repeated addition. To prepare for multiplication, describe repeated addition calculations in terms of how many 'lots of' or how many equal groups, before introducing the <math>\times</math> sign. Songs are also useful.</p>	<p>Give real life problems involving addition of two each time and use apparatus to represent and solve in lots of different ways, e.g. socks, money etc.</p>    <p>Place counters/cubes/ Numicon twos over top of a 100 square. Write or find the repeated addition to match. How many 'lots of' 2?</p> <p>Combine equipment in part whole models and bar models. Explore which are the parts and which is the whole. Predict the whole amount. Give the whole amount and see how many equal groups of two can be made.</p>   <p>True or false - is <math>2 + 2 + 2 = 3</math> lots of 2? Explain.</p> <p>Place twos along the counting stick or number track/number line and mark on the multiples of two. Hide one number for the children to 'guess', explaining how they know. Remove multiples and use to practise counting forwards/backwards from different starting points and to find missing numbers.</p> <p>When children are confident with the concept of repeated addition and describing 'lots of', the multiplication sign can be introduced as a quicker way to express this idea.</p>	<p>Colour to show the pattern. What do you notice?</p>  <p>How many twos are in 20? Circle lots of twos to find out. Show the repeated addition. Circle twos to find a total.</p> <p>Discuss relationships - Which is more - 3 lots of 2 or 5 lots of 2? How do you know? Represent repeated addition visually alongside practical representations. Number sentences could be matched to visuals and visuals to other visuals showing the same calculation. When children are confident with one way of visually recording, begin to encourage them to record practical work in different ways.</p> <table border="1" data-bbox="1518 895 2040 1007"> <tr> <td colspan="4">In an array</td> <td colspan="4">On a bar model</td> </tr> <tr> <td colspan="4">XXXX</td> <td colspan="4">8</td> </tr> <tr> <td colspan="4">XXXX</td> <td>2</td> <td>2</td> <td>2</td> <td>2</td> </tr> </table>  	In an array				On a bar model				XXXX				8				XXXX				2	2	2	2
In an array				On a bar model																						
XXXX				8																						
XXXX				2	2	2	2																			

## 2x Table: Investigating Relationships (Scaling - Doubles Link).

Notes	Concrete	Visual to Support Abstract
<p>Children should begin to recognise that counting in twos can mean lots of 2 or double the amount/twice as many.</p>	<p>Solve real life problems involving doubles using a range of concrete apparatus.</p> <p>Use arrays to reinforce the link between counting in twos and doubling.</p> <p> When introducing arrays, make one lot of 2, then two lots of two/2+2 etc so children see the pattern that emerges. Ask them to make the next line in an array and describe it (e.g. or 2+2+2 or three lots of two). Can they predict what the next number sentence will be?</p> <p>Eg. Laura thinks this shows <math>2 + 2 + 2 + 2 + 2</math> but Ravi says it shows <math>5+5</math>. Who is right? Show me with cubes, a tens frame, counters etc.</p> <p>Children can make their own arrays using counters to show <math>2+2+2</math> and double 3. What do they notice?</p> <p>Tens frames are also useful to demonstrate the link and for children to begin drawing their own arrays.</p> <p></p>	<p>True or false: 2 lots of 5 = 5 lots of 2? Draw an array on a tens frame to prove it.</p> <p>Group spots or crosses within an array to show double 5. Now draw around a duplicate of the array to show 5 lots of 2. Write a number sentence to go with each.</p> <p>Does this number line show 5 lots of 2 or 2 lots of 5? Explain how you know.</p> <p></p> <p>Could you draw a number line to represent 2 lots of 5?</p> <p>Match number sentences to visuals or concrete representations.</p>

## Counting in Tens- Relationship Between Repeated Addition and Skip Counting.

Notes	Concrete	Visual to Support Abstract
<p>Activities are broadly similar to counting in twos. The emphasis is on children becoming confident with the concept of repeated addition and ten times facts, so use the term 'lots of' as well as the x sign, to encourage understanding.</p>	<p>Give real life problems involving repeated addition of ten and use concrete materials to represent and solve. Match apparatus to repeated additions and vice versa. How else could we say it? <math>10+10+10+10</math> or 4 lots of 10.</p>  <p>Place tens horizontally across 100 square to show relationship. On empty 100 square, what would 4 tens come to? How many tens are in 40? Prove it.</p>   <p>Put tens along a counting stick or number line and mark on the multiples of ten. Discuss which numbers would be in between them. Jump in tens along a numbered number line and count up by skip counting on a counting stick. (See page 7). You may want to use a bead bar to begin with, so children can physically see the ones within the tens.</p> <p>True or false: <math>10 + 10 + 10 = 3</math> lots of 10? Make something to show me.</p>	<p>Visual to Support Abstract</p>  <p>Colour to show the multiples of ten or to solve problems. E.g. colour to find <math>10+10+10</math>.</p> <p>Write a repeated addition or find a different visual which shows the same calculation. Give out cards with multiples of ten on them. Muddle them up and ask children to reorder and place on an empty number track or line. Which numbers would go in between them? Draw number lines to solve repeated addition or 'lots of' facts. E.g. Draw 4 lots of ten on the number line.</p> 

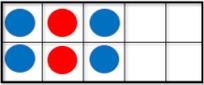
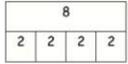
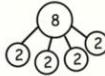
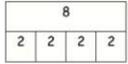
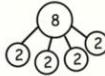
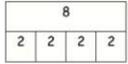
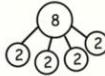
## 10x Table- Investigating Relationships

Notes	Concrete	Visual
<p>Activities broadly similar to that of 2x table, as emphasis is on children becoming confident with multiplication facts.</p>	<p>Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use to discuss the commutative property.</p> <p>E.g. Discuss how this can show both <math>10 \times 4</math> and <math>4 \times 10</math> (<math>10+10+10+10</math> or <math>4+4+4+4+4+4+4+4+4+4</math>). Match repeated addition and 'lots of' and multiplication calculations to concrete representations and to each other.</p> 	<p>Draw arrays to match repeated additions and vice versa.</p> <p>Circle counters/crosses in arrays to show the number sentence given.</p>

## 5x Table- Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract																																																																																																				
<p>Activities broadly similar to counting in twos, as emphasis is on children becoming confident with the concept of repeated addition.</p> <p>See page 21 for introducing arrays.</p>	<p>Use a range of practical apparatus to 'act out' and represent problems.</p> <p>Write the repeated addition to match a representation.</p> <p>Use 10s frames to help make links between 5s and 10s.</p> <p>Place cubes in fives on top of a hundred square to show the pattern.</p> <p>Put practical equipment which represents fives (e.g. 5p coins/cubes etc) in part whole models and bar models. Discuss which are the parts and which is the whole. Use to solve repeated addition calculations and also describe in terms of 'lots of.'</p> <p>Make arrays to show different repeated addition calculations. Use to discuss the commutative property.</p> <p>E.g. Discuss how it can show both 5 lots of 4 and 4 lots of 5, <math>5+5+5+5</math> or <math>4+4+4+4+4</math>. Match calculations to concrete representations and vice versa.</p> <p>True or false - Is 5 lots of 3 = 3 lots of 5? Explain how you know.</p> <p>Place concrete apparatus along a counting stick and discuss the relationship between repeated addition and skip counting.</p> <p>Jump in fives along a counting stick and count up by skip counting. (See 'The Counting stick', page 7).</p> <p>Make two sticks of ten and break them in half. How many fives have you got? What do you notice? What do you think will happen with three tens?</p>	<p>Colour in the 100 square to show patterns and make the link between fives and tens visually.</p> <table border="1" data-bbox="1442 411 1697 619"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table> <p>Jump in fives along a numbered number line and count up by skip counting. Match different visual representations (array, bar model, part whole model, number line) to repeated addition and multiplication calculations or calculations to different visuals.</p> <p>True or false: When I count in fives, the numbers I say will always have a five in the ones?</p> <p>Sort cards with 2 digit numbers on them according to whether or not they are a multiple of five, checking with concrete apparatus. Explain what you notice. Predict other numbers that you would say when counting in fives. Put cards on 100 square to check.</p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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## 2x Table: Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract				
<p>Make the point that reading and recording repeated addition calculations is time consuming, so it can be referred to 'lots of'. The multiplication sign can be shown to represent the 'lots of'.</p> <p>It is important to investigate the relationships between multiples when skip counting.</p>	<p>Give real life problems involving repeated addition of two and use concrete materials to act out the problem and solve. Draw to show.</p> <p style="text-align: center;"><math>2+2+2=</math></p>  <p>Match apparatus to repeated addition statements and then multiplication statements and vice versa.</p> <p>Place concrete apparatus in twos along a counting stick and mark on the multiples. Point to a multiple and ask the children to write the repeated addition then the multiplication.</p>  <p style="text-align: center;">Get out coins to represent repeated addition and multiplication calculations.</p> <p>How much money is there? Write the repeated addition calculation and the multiplication statement. How much would there be if I added 2p more? Spent 2p? etc. Find out.</p> <p>Place cubes over numbers on 100 square to match calculations then use them to solve calculations.</p> <p style="text-align: center;">Use part whole and bar models and discuss which are the parts and which is the whole.</p> <p>Put twos in the parts. Describe the parts and the whole in a number sentence. Now show the whole amount. How many groups of two will you get? Match and later write repeated addition, multiplication and division calculations to match.</p>	<p>Once children are secure with one way to visually represent, explore different ways. E.g. show 3 lots of 2 as a repeated addition, practically and on a 100 square, number track or number line, in an array.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Show 4 lots of 2 as</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;"> <p>an array</p> <p>XX XX XX XX</p> </td> <td style="width: 50%; padding: 5px;"> <p>a bar model</p>  </td> </tr> <tr> <td style="padding: 5px;"> <p>a part whole model</p>  </td> <td style="padding: 5px;"> <p>a number line</p>  </td> </tr> </table> </div> <p>Draw pictures and use visuals to represent and help expose the relationships within multiplication problems and the link to division.</p> <p>Match different representations to calculations and find visual representations which show a given calculation. Make up problems to represent different visuals or calculations.</p> <p>Draw visual representations to solve problems. These may begin as simple pictures then move onto number lines, part whole models etc.</p> <p>Ensure examples include multiplying by 1 and 0.</p>	<p>an array</p> <p>XX XX XX XX</p>	<p>a bar model</p> 	<p>a part whole model</p> 	<p>a number line</p> 
<p>an array</p> <p>XX XX XX XX</p>	<p>a bar model</p> 					
<p>a part whole model</p> 	<p>a number line</p> 					

After lots of practical experiences, show how the x sign can be used as a quicker way to express 'lots of'. (If children are confusing the multiplication and the addition sign, use x to cross out 'lots of') as a prompt for those confusing the orientation.

Use the counting stick methods (see page 7) to explore relationships between multiples of 2 and find more efficient ways to derive them using underlying skills and known facts. What do you think  $11 \times 2$  would be?  $12 \times 2$ ? How do you know?

Once confident with the counting stick, the teacher can model relationships and missing numbers on a number line.

The counting stick can also be used for the reading scales link:



Give out number cards from 1 to 20. Use cubes to put numbers into twos. What do you notice?

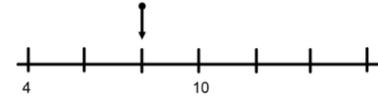
Colour to show the pattern of the two times table. What are the numbers you haven't coloured called? What are the multiples of two called? What do you notice about each?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

How many twos did you count to reach 10?

How many do you think you will count to reach 20?

Where would 14 go? What about 13?

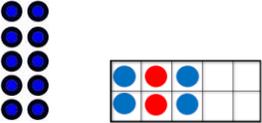
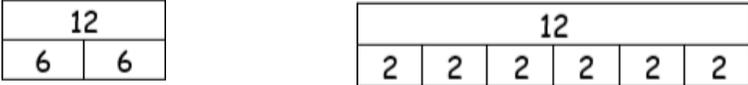
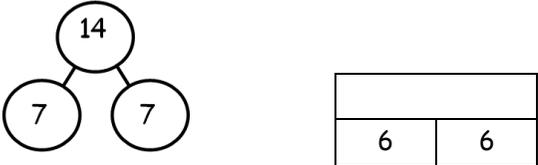
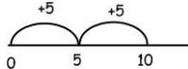


Sort the numbers cards which could and couldn't be put into twos. Predict which numbers in the 20s could be put into twos etc.

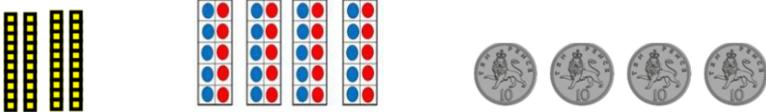
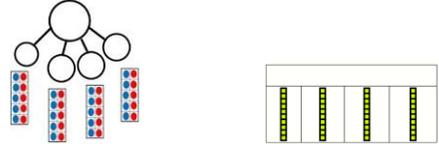
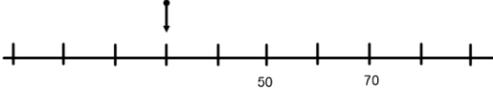
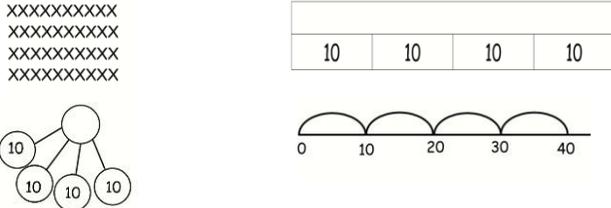
Reason to address misconceptions or help make generalisations.

- Count in twos to solve this:  $2+2+2+2=$
- What can help you solve this:  $2+5+2+2=$
- True or false:  $9 \times 2 < 4 \times 2$ ? Explain how you know.
- Look at these numbers: 10, 20, 30, 40. Which can be put into twos and which can't? Use concrete apparatus to find out.
- Which is the odd one out: 23, 18, 17? Explain why.

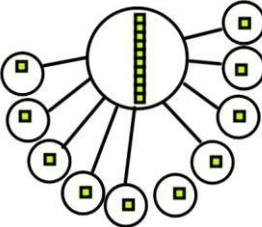
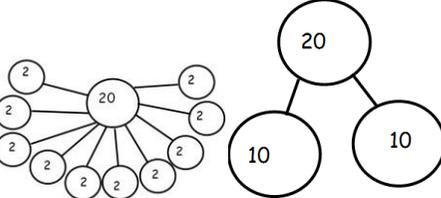
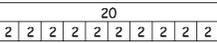
## 2x Table: Investigating Relationships (Scaling - Doubles Link).

Notes	Concrete	Visual to Support Abstract
<p>Link <math>\times 2</math> to doubling, as doubling is needed when multiplying larger numbers by 2 (e.g. <math>34 \times 2</math>). Teach doubles before <math>\times 2</math>, so the link can be made. Ensure that some problems involve scaling up by 2 (doubling) as well as counting in twos, using language such as twice as much, double and two times (E. <math>3 \times 2</math> can mean 3 two times or 3 lots of 2).</p>	<p>Use apparatus to represent and solve doubles problems.</p>  <p>Investigate relationships between skip counting in twos and doubling by making arrays (see page 21).</p> <p>Match calculations to concrete representations and use apparatus to show calculations in different ways. E.g. show it with coins, cubes etc.</p> <p>True or false- 2 lots of 5=5 lots of 2. Why? Prove it. What would <math>2 \times 6</math> look like on a part whole model/bar model? What about <math>6 \times 2</math>?</p> <p>Use concrete apparatus to make an array to show a given calculation. Write 2 repeated addition and 2 multiplication calculations (and later division) for an array made by your partner.</p> <p>Identify the repeated addition to match a problem. e.g. Does a problem require <math>4+4</math> or <math>2+2+2+2</math>?</p> <p>Given concrete apparatus, make up a problem that it could represent.</p>	<p>Represent problems and calculations visually by drawing circles or crosses in an array, bar models and part whole models. E.g. show <math>2+2+2+2+2+2</math>, now show double 6. Explain what you drew each time. Write multiplication calculations to match. What's the same? What's different?</p>  <p>Record and solve problems and calculations visually in a range of ways. Write a repeated addition, multiplication, division calculation to match each visual.</p>  <p>Write problems to match calculations and visuals. Explore associated language, making clear that <math>\times 2</math> can mean lots of or two times as many.</p>
<p>Reasoning to address misconceptions or help make generalisations.</p>	<ul style="list-style-type: none"> <li>Show 7 lots of 2 on a number line/bar model/part whole model. Now show 2 lots of 7. What's the same? What's different?</li> <li>Does the number line show 5 lots of 2 or 2 lots of 5? Explain.</li> </ul>  <p>Always, sometimes, never: double <math>7 = 7 \times 2</math>? Double <math>43 = 43 \times 2</math>?</p>	

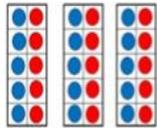
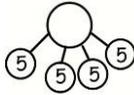
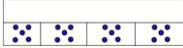
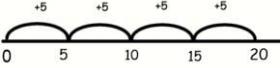
# 10x Table: Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Make the point that recording and reading repeated addition calculations is time consuming, so we can refer to them as lots of 10.</p> <p>Count past 100 and show what happens when we reach 10 lots of ten.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Give real life problems involving repeated addition of ten and use apparatus to act out the problem and solve.</p>  <p>Write it as a repeated addition. How else could we say it? <math>10+10+10+10</math> or 4 lots of 10.</p>  <p>Place tens horizontally across a 100 square to show the relationship. On an empty 100 square, what number would 4 tens come to? How many tens are in 40? Show on a 10x10 peg board.</p> <p>Put out cards with multiples of ten on them. Muddle them up for children to reorder and place on an empty number track or line. Which numbers would go in between the multiples of ten? Check with apparatus.</p> <p>Put apparatus into part-whole models and bar model and discuss which are the parts and which is the whole. Describe them using repeated addition and multiplication calculations. What would the division calculation be?</p>  <p>Use counting stick methods (see page 7) to explore the relationships between multiples of 10 and find more efficient ways to derive them using underlying skills and known facts. What is <math>11 \times</math>, <math>12 \times</math>? How do you know?</p>	<p><b>Visual to Support Abstract</b></p>  <p>Colour to show calculations, e.g. <math>2 \times 10</math>. Write the repeated addition and multiplication calculations to match.</p> <p>What number is this pointing to? How could you find out?</p>  <p>Record one calculation in a range of ways - an array, part whole model, bar model, number line etc.</p>  <p>What does the part whole model show? Find a different visual that shows the same calculation. Match calculations to visuals and vice versa. Use different representations to show or solve a problem. Write a problem to show a calculation.</p>

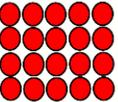
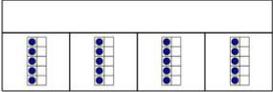
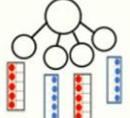
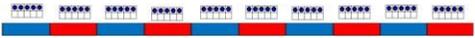
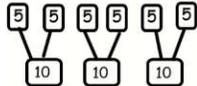
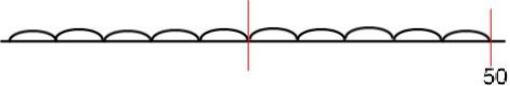
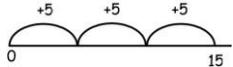
## 10x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract				
<p>It is important to investigate, not just 10 lots of something, but also idea of scaling by making 10x greater as this is fundamental to the later understanding of place value.</p> <p>Use language related to scaling, e.g. 10 times as much/many, 10 times greater and give practical problems including these.</p>	<p>Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use them to discuss the commutative property. E.g. Discuss how this can show both <math>10 \times 4</math> and <math>4 \times 10</math> (<math>10+10+10+10</math> or <math>4+4+4+4+4+4+4+4</math>).</p> <p>Match calculations to concrete representations.</p>  <p>Extension: Discuss the relationships that exist whilst working with practical equipment. What else could this represent? E.g. 2 lots of <math>2 \times 10</math> or <math>(2 \times 10</math> and <math>2 \times 10)</math> or 4 lots of <math>2 \times 5</math> or 1 lot of 10 and 3 lots of 10 etc.</p>  <p>Investigate what happens when we make ones 10x bigger. Also, show on bar models.</p> <p>Write the repeated addition. Write the multiplication. What would the whole be if we put 2 in each part? How about 3? etc.</p> <p>What division calculation would this show?</p>	<p>Match pictures of arrays to calculations and vice versa. Draw arrays, part whole models and bar models to match calculations and to solve calculations and problems.</p> <p>Look at the visuals. What's the same? What's different?</p>   <table border="1" data-bbox="1563 1050 1711 1129"> <tr> <td colspan="2">20</td> </tr> <tr> <td>10</td> <td>10</td> </tr> </table> <p>Write a word problem to go with a visual or calculation.</p>	20		10	10
20						
10	10					
<p>Reasoning to address misconceptions or help make generalisations.</p>	<p>What do you notice about the multiples of ten? Which is the odd one out? Why?            10, 18, 20, 40, 110.  <math>10 \times 4 &gt; \underline{\quad}</math> What could it be? What couldn't it be?            True or false: <math>10 \times 3 &lt; 3 \times 10</math>. Explain how you know.</p>					

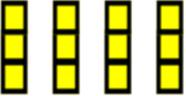
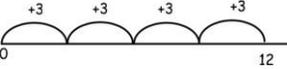
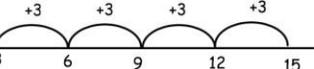
## 5x Table: Relationship Between Repeated Addition and Skip Counting

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<p>Make the point that recording and reading repeated addition calculations is time consuming, so we can refer to them as 'lots of' 10.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Give real life problems involving repeated addition of five and use apparatus to act out the problem and solve. Draw to show understanding.</p>  <p>Write a repeated addition to show this. How else could we say it? <math>5+5+5+5+5=</math> or <math>6 \times 5</math>. Can you show this with concrete equipment, e.g. coins?</p> <p>Place cubes in fives over a 100 square. What do you notice? Place cubes along a counting stick and count in fives.</p>  <p>Model and write repeated addition and multiplication. Show how these can be represented as jumps on a number line.</p> <p>Use counting stick methods (see page 7) to explore the relationships between multiples of 5 and to find more efficient ways to derive them using underlying skills and known facts. What do you think <math>11 \times 5</math> would be? <math>12 \times 5</math>? How do you know? Make sure the relationship of <math>\times 9</math> to tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. How could you find <math>9 \times 3</math>? <math>9 \times 4</math>? The counting stick can also be linked to reading scales in fives and finding missing numbers. What is the missing number? How did you find it? Did anyone find it a different way?</p> 	<p>Represent repeated addition, multiplication and division calculations in a variety of ways (see examples for <math>\times 2</math> on page 30). Match calculations to arrays, bar models, part-whole models and number lines and vice versa.</p>   <p>Write the repeated addition and multiplication that these show. Write the 2 division calculations that they show. Use to find missing numbers, e.g. <math>\_\_\_ \times 5 = 20</math>.</p> <p>Write a problem that the visual shows or to match a calculation.</p> <table border="1" data-bbox="1420 794 1715 1007"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table> <p>Colour a 100 square to show the pattern of fives. What do you notice?</p> <p>Record and solve calculations and problems on an empty number line.</p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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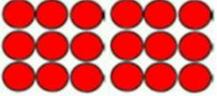
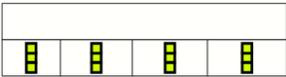
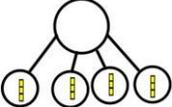
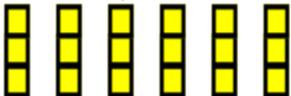
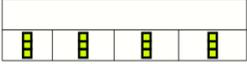
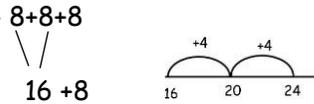
## 5x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract
<p>It is important to exploit doubling and halving knowledge to investigate the relationship between the fives and the tens. Use language related to scaling, e.g. 5 times as much/many, 5 times greater and give practical problems including these.</p> <p>When looking at multiplication, ask what it could mean. E.g. <math>5 \times 6</math> could be: 5 lots of 6, 6 lots of 5, 5 six times, 6 five times, 6 multiples of 5, 5 multiples of 6, <math>5+5+5+5+5</math>, <math>6+6+6+6</math> etc.</p>	<p>Make arrays (see page 21) to show different multiplication/repeated addition and division calculations. Use these to illustrate the commutative property. E.g. Discuss how it can show both <math>5 \times 4</math> and <math>4 \times 5</math>, <math>5+5+5+5</math> or <math>4+4+4+4</math>. Match calculations to concrete representations.</p> <p> Extension: What else could it show? (<math>2 \times 10</math>, 3 fives and another five). Show me two fives and another two fives. How many fives have you got?</p> <p> </p> <p>Investigate the commutative property. Make a bar model to show <math>4 \times 5</math>. Now make one to show <math>5 \times 4</math>. What's the same and what's different? Make up a problem for each.</p> <p>Investigate the relationships between tens and fives practically and combine in part-whole models and bar models.</p> <p>Play as a game. Show 3 lots of 10. Predict how many lots of 5? How many will there be in 4 tens? Prove it using cubes, tens frames and other number equipment. Show it on a 100 square.</p> <p>2 lots of 10.  How many lots of 5? </p> <p>Explore the relationship between <math>10 \times</math> and <math>5 \times</math> on a counting stick, putting concrete apparatus along the stick.</p> <p></p> <p>If <math>10 \times 5 = 50</math>, what is <math>5 \times 5</math>? What if it were <math>10 \times 2</math>? <math>10 \times 10</math>?</p>	<p>Draw and group arrays to show different multiplication and repeated addition calculations. E.g. <math>5 \times 4</math>, <math>4 \times 5</math>, <math>5+5+5+5</math> or <math>4+4+4+4</math>. Also, link arrays to division. Make the link between the fives and the tens.</p> <p> <math>3 \times 10 = \underline{\quad} \times 5</math>.</p> <p><math>10 \times 5 = 50</math>, what is <math>5 \times 5</math>?</p> <p></p> <p>What tables fact does the number line show? How can it help you to find <math>6 \times 5</math>? What do you think <math>12 \times 5</math> would be?</p> <p> This shows <math>3 \times 5</math>. Predict <math>6 \times 5</math>. Draw a number line to show <math>3 \times 5 + 3 \times 5</math>.</p> <p>True or false?  <math>6 \times 5 &gt; 2 \times 5</math>.  <math>4 \times 5 &gt; 4 \times 2</math>. Explain how you know.</p> <p>How many different ways can you find the answer to <math>8 \times 5</math>? (Discuss the different ways to help children understand the relationships within multiplication tables, e.g. count in fives, or <math>8+8+8+8</math> using doubles and addition, or ten fives subtract two fives or double four fives).</p>

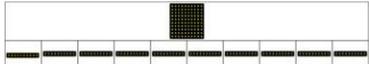
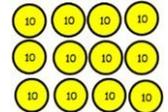
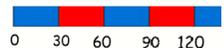
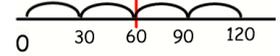
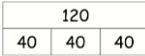
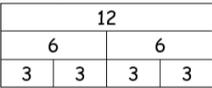
### 3x Table: Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Although the 3x table is covered in Year 3, counting in threes is covered in Year 2 and there have been questions on Year 2 SATs papers testing the 3x table. Therefore, it makes sense to cover it in Year 2 and revise it in Year 3.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Represent and solve problems involving repeated addition of three using concrete apparatus.</p>  <p>Write a repeated addition to show this. How else could we say it? <math>3+3+3+3 =</math> or <math>4 \times 3 =</math></p> <p>Can you show this with cubes? Place cubes/number pieces/Cuisenaire rods along a counting stick and mark in threes.</p>  <p>Use counting stick methods (see page 7) to explore the relationships between multiples of 3 and find more efficient ways to derive them using underlying skills and known facts.</p> <p>What do you think <math>11 \times 3 = ?</math> <math>12 \times 3 = ?</math> How can <math>10 \times 3</math> help you find <math>5 \times 3</math>? <math>9 \times 3</math>?</p> <p>Make sure the relationship between the ninth and tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. Could you use it for <math>9 \times 4</math>? <math>9 \times 6</math>?</p> <p>Model and write repeated addition and multiplication calculations. Begin to derive the division facts using counting stick. E.g. How many threes in 15?</p> <p>Use an empty counting stick to link facts together and revise other tables. E.g. Point to the tenth multiple and ask if this is 30, what would <math>9 \times</math> be? What about if the tenth multiple was 50? 100? 20?</p>	<p><b>Visual to Support Abstract</b></p>  <p>Colour a 100 square to show the pattern of threes. Can you spot any patterns?</p> <p>Use counting stick relationships to find missing numbers on empty number lines. Draw number lines to solve calculations and problems.</p> <p>What tables fact does the number line below show? Can you use this to find <math>8 \times 3</math>? Rory says it also shows <math>2 \times 3 + 2 \times 3</math>. Is he right?</p>  <p>Match repeated addition and multiplication calculations to number line visuals and vice versa. Make up a problem that a calculation or visual could show.</p> <p>Gemma drew this number line to show <math>4 \times 3</math>. What is her mistake?</p> 

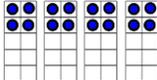
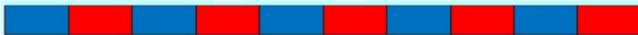
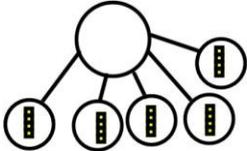
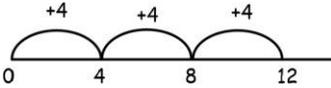
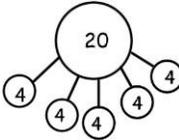
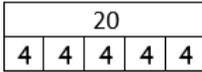
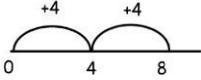
### 3x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract				
<p>Use language related to scaling as well as skip counting. E.g. <math>4 \times 3</math> can be 4 lots of 3 or 4 three times or triple four.</p> <p>Tripling is important, as it helps to show the commutative property and encourages children to think about tables fact from different perspectives.</p>	<p>Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use them to discuss the commutative property. E.g. Discuss how it can show both <math>6 \times 3</math> and <math>3 \times 6</math>, <math>6+6+6</math> or <math>3+3+3+3+3+3</math>. Match calculations to concrete representations and to visuals. What division facts can you work out?</p>  <p>Extension: What else could it show? (<math>2 \times 9</math>, <math>9 \times 2</math>). Show me threes threes and another three threes. How many threes have you got? How could we write a calculation to show this?</p>   <p>Make a bar model/part whole model to show <math>4 \times 3</math>. How many parts will you have? How many in each part? What will the whole be? Now make one to show <math>3 \times 4</math>. What's the same? What's different? Can you make up a problem for each representation? Can you use the bar model above to help you work out <math>8 \times 3</math>? <math>2 \times 3</math>? What would your bar model look like if you wanted to show 4 three times/three times as many?</p> <p>Investigate relationships practically with discussion and prediction of what will happen when numbers are changed, doubled etc.</p> <p>Combine in part-whole models and bar models practically.</p>  <p>Show 6 lots of 3. Predict 12 lots of 3, 3 lots of 3? Prove it using concrete equipment. Show it on a 100 square/counting stick/number line.</p>	<p>Show <math>3 \times 4</math></p> <table border="1" data-bbox="1411 338 2027 491"> <tr> <td>In an array</td> <td>In a bar model</td> </tr> <tr> <td>In a part whole model</td> <td>On a number line</td> </tr> </table>   <p>Use visuals to solve missing number problems and investigate the relationship between multiplication and division. Write multiplication and division sentences represented by visuals.</p> <p>What do you notice: <math>2 \times 3 = 6</math> <math>4 \times 3 = 12</math></p> <p>Which calculation would come next? Explain. Show a calculation. What could it mean? E.g. <math>3 \times 8 - 8</math> lots of 3, 3 lots of 8, 8 three times, 3 eight times, 3 multiples of 8 etc. Discuss different ways to solve. Practise tripling, which makes use of doubling and bridging.</p> <p>E.g. e.g. <math>3 \times 8 - 8 + 8 + 8</math></p> 	In an array	In a bar model	In a part whole model	On a number line
In an array	In a bar model					
In a part whole model	On a number line					
Reasoning	<ul style="list-style-type: none"> <li>• True or false: <math>6 \times 3 &gt; 2 \times 3</math>. Explain how you know.</li> <li>• <math>3 \times 10 &gt; \underline{\quad}</math>. What could go there? What couldn't go there?</li> <li>• <math>3 \times 3 \underline{\quad} 7 \times 3</math>? Which of these could not go in the middle: <math>5 \times 3</math>, <math>3 \times 4</math>, <math>3 \times 2</math>. Explain how you know.</li> </ul>					

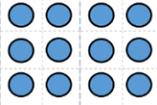
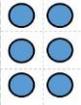
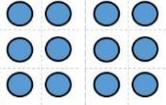
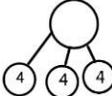
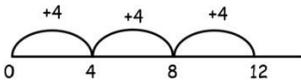
Investigating the Distributive and Associative Law/Working with Larger Numbers (These activities can take place with each table once the concept is secure, to deepen understanding of the properties and application of multiplication).

Notes	Concrete	Visual to Support Abstract
<p>Begin to investigate the distributive (<math>4 \times 5 + 4 \times 5 = 8 \times 5</math>) and associative (<math>2 \times 4 \times 5</math>) properties of multiplication informally during practical work.</p>	<p>In pairs, make an array of <math>4 \times 10</math>. Think about the different ways it can be put into equal groups and how to record.</p>  <p>E.g. Some may find <math>2 \times 20</math>, <math>4 \times 10</math>, <math>8 \times 5</math>. Can they split these further into <math>2 \times 20</math> which is the same as 2 lots of 2 tens; or <math>4 \times 10</math> which is the same as 4 lots of 2 fives etc. This could be explained as <math>2 \times 10 + 2 \times 10</math> or <math>4 \times 2 \times 5</math>. Children may record in words (e.g. 2 lots of 2 tens or 2 tens + 2 tens) or just discuss this to explore the concept.</p>	<p>Draw the arrays and circle the different groups you have found. Write calculations.</p> <p>Draw arrays and number lines to represent calculations based around the distributive law, e.g. <math>3 \times 3 + 2 \times 4</math>. Can they make up a problem it could represent?</p> <p>When confident, begin to think about how to visually represent the associative law e.g. <math>2 \times 5 \times 2</math> (or 2 lots of <math>5 \times 2</math>).</p>
<p>Use previous strategies and visuals to help understand multiplication and division with larger numbers and fractions.</p> <p>When working with hundreds, also refer to them as the number of tens. E.g. 300 or 30 tens (<u>300</u>).</p>	<p>Explore the effect of multiplying by 10 on part whole/bar models. The bar model shows <math>10 \times 10 = 100</math>. Use equipment to find <math>20 \times 10</math>.</p>  <p>Predict <math>30 \times 10</math>, <math>40 \times 10</math> etc.</p> <p>What do you notice? What about <math>11 \times 10</math>, <math>23 \times 10</math> etc.</p> <p>Use tens counters (or imagine the ones counters are tens) to make the link between known facts and new facts. I know <math>3 \times 4 = 12</math>, so what else do I know? <math>30 \times 4</math>, <math>4 \times 30</math>, <math>40 \times 3</math>, <math>3 \times 40</math>. Find associated division facts. Represent these new facts using apparatus that links back to previous work and known facts each time.</p>    <p>Explore with cuisenaire rods/number pieces. If each number piece is equivalent to one whole, how many quarters in 1? 2? 3? Mark a counting stick in ones and together mark on quarters. How many quarters will be in 5 whole ones? What do you notice?</p>	<p>Explore place value relationships on number lines and link back to single digits.</p>    <p>Bertie thinks he can show both the bar models above with an array made out of tens place value counters. Is he right?</p> <p><u>Link to fractions:</u> Make the link between quarters and dividing by four explicit. E.g. Use cubes with bar models to quarter whole numbers by halving and halving again. Give one quarter on a bar model and use it to find the whole amount (double and double again). Also, reinforce this strategy when working with other fractions where four parts are needed e.g. <math>4/5</math>, <math>4/8</math> of a quantity.</p> 

## 4x Table: Relationships - Skip Counting

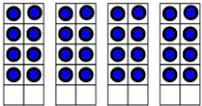
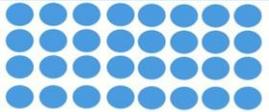
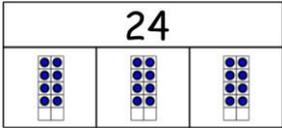
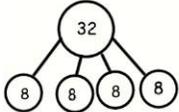
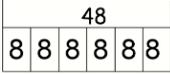
Notes	Concrete	Visual to Support Abstract
<p>Revise repeated addition so secure.</p> <p>Show calculations and ask what they could mean, e.g. <math>3 \times 4</math> could be 4 lots of 3 or 4 three times, 3 lots of 4 or 3 four times, 4 multiples of 4, 4 multiples of 3, <math>3+3+3+3</math> or <math>4+4+4</math>.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Represent and solve problems involving multiplication of four using different types of concrete apparatus. Match representations to calculations and vice versa.</p> <p> <math>4 + 4 + 4 + 4 =</math> <math>4 \times 4 =</math></p> <p></p> <p>Use counting stick methods (see page 7) to explore the relationships between multiples of 4. Ensure discussion of <math>11 \times 4</math> and <math>12 \times 4</math> is included and focus upon <math>\times 9</math> and <math>\times 8</math> particularly, discussing whether this strategy would work for other multiples. E.g. Could you use it for <math>9 \times 8</math>? (E.g. 80-8).</p> <p>Write down numbers below 40. Which are multiples of 4? Predict which definitely cannot be multiples of 4. Explain why. Prove it.</p> <p>Combine apparatus into part whole models/bar models. Explore which are the parts and which is the whole. How could we say this? (5 lots of 4 or 4 five times).</p> <p></p> <p>Match apparatus to multiplication and division calculations and vice versa.</p> <p>Use them to solve missing number calculations practically. E.g. <math>20 \div \_ = 4</math>.</p> <p>Use concrete apparatus to solve <math>\_ \times \_ = 40</math>. What could it be? What couldn't it be? Revise link between tens and fives (e.g. <math>4 \times 10</math> so how many fives?)</p>	<p>Draw empty number lines to solve problems.</p> <p></p> <p>Colour the pattern of fours on a 100 square. Discuss. What do you notice? Sort numbers. Which cannot be multiples of 4? How do you know?</p> <p> </p> <p>Write the multiplication and division statements shown by the completed visuals. Use visuals to find missing numbers and solve problems. E.g. <math>\_ \times 4 = 28</math>. <math>\div 4 = 9</math></p> <p>Write a problem that the visual could represent.</p> <p> If this shows <math>5 \times 4</math>, what would <math>10 \times 4</math> be?</p> <p> What tables fact does this show? How can it help you find <math>4 \times 4</math>?</p> <p><math>\_ \times \_ = 40</math>. What could it be? What couldn't it be?</p> <p>Use what is known about multiples of 1, 2, 3, 4, 5, 10 to make predictions. Encourage systematic work.</p>

## 4x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract						
<p>These strategies rely heavily on the ability to double so this must be secure.</p> <p>See page 21 for introducing arrays.</p> <p>It is important to see the link between <math>\times 4</math> and doubling early, as doubling is more efficient than skip counting, especially for later work with larger numbers or decimals.</p> <p>Include problems which involve scaling contexts, e.g. 4 times greater, 4 times as many/much and draw visuals to show these.</p>	<p> What calculation does the array show? Asif says this shows <math>3 \times 4 = 12</math> but Alex says it shows <math>4 \times 3 = 12</math>. Who is right? Explain.</p> <p>Can you use this array to help you work out <math>6 \times 4</math>? What do you think <math>12 \times 4</math> would be? Play a game: show an array and use it to predict new tables facts.</p> <p>Link 2x table to 4x table. Make me an array to show <math>2 \times 3</math>. What do you think <math>4 \times 3</math> would be? Why? Make an array to check.</p> <p> </p> <p>Make or draw arrays to show different multiplication calculations from 4x table. What facts from 2x table would they show? What do you notice?</p> <p>Winston says any multiple of two will also be a multiple of four. Is he right? How could you find out?</p> <p>Discuss the relationship with division when working and make links to division calculations and problems. (See page Inverse Relationships, pages 11-14).</p> <p>Sort given multiplication calculations according to whether they are correct or incorrect. Explain how you knew, (e.g. Explain why <math>6 \times 4 = 23</math> couldn't be correct). Use this as an opportunity to revise previous tables.</p>	<p>Write calculations to match arrays and draw arrays to match calculations.</p> <p>Look at word problems and find the appropriate multiplication calculation to represent it. Make up a different word problem for the same calculation.</p> <p>Draw arrays/bar models/number lines to show facts from the two's times table. Use them to predict facts from the 4x table.</p> <p> E.g. <math>2 \times 4 = 8</math>, so <math>4 \times 4 = 16</math>.</p> <p>On a multiplication grid, record the first 5 facts from the 2 times tables and then repeat for the 4 times table. What do you notice?</p> <p>Represent multiplication and repeated addition statements visually, e.g. show <math>3 \times 4</math> in four different ways. Write multiplication and division calculations to match and match visuals to calculations.</p> <p>Draw visuals to solve missing problems. E.g. <math>3 \times 4</math>. <math>\_\div 4 = 6</math>.</p> <p> </p> <p><table border="1" data-bbox="1339 1225 1496 1279"> <tr><td></td><td>12</td><td></td></tr> <tr><td>4</td><td>4</td><td>4</td></tr> </table> </p>		12		4	4	4
	12							
4	4	4						

		<p>When showing tables facts, ask for all the different interpretations of what they could mean. E.g. <math>7 \times 4</math> could mean 7 multiples of 4, 7 lots of 4, 7 four times, 4 lots of 7, 4 multiples of 7, 4 seven times etc. Discuss different strategies to work it out, drawing on strategies already known to reduce memory load and access a wider range of facts. E.g. <math>4 \times 7</math> could be solved by thinking of it as four sevens and using doubling skills.</p> $  \begin{array}{c}  7+7+7+7 \\  \underbrace{\quad\quad}_{14} \quad \underbrace{\quad\quad}_{14} \\  \underbrace{\quad\quad\quad\quad}_{28}  \end{array}  $
<p>Reason to address misconceptions or help make generalisations.</p>	<ul style="list-style-type: none"> <li>• True or False: <math>9 \times 4 &gt; 4 \times 3</math>. Explain how you know.</li> <li>• <math>5 \times 4 &lt; \underline{\quad}</math> What could go there? What couldn't? Why.</li> <li>• True or false: I can find the product of 5 and 4 by getting 4 lots of 5?</li> <li>• Show 7 lots of 4 on a number line/bar model/part whole model. Now show 4 lots of 7. What's the same? What's different?</li> <li>• Sophie says, "<math>2 \times 2 \times 2 = 6</math>" but Bartosz says she is wrong because "<math>2 \times 2 \times 2 = 8</math>" who is right. Prove it with concrete apparatus. Prove it with an array.</li> </ul>	

## 8x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Revise and make clear the links to prior learning. Children should already know all facts from <math>0 \times 8</math> to <math>5 \times 8</math> from previous work and be able to quickly find <math>\times 9</math>, <math>\times 11</math>, <math>\times 12</math>. This should be discussed and made explicit. The only new facts are <math>6 \times 8</math>, <math>7 \times 8</math>, <math>8 \times 8</math>.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Use different types of concrete equipment to solve problems and represent repeated addition, multiplication and division calculations. Write calculations to match concrete representations.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <math>8+8+8+8</math>  <math>4 \times 8</math> </div> </div> <p>Use cubes/counters to make arrays in eights. Write the multiplication and division statements to go with them. Read out problems and in pairs make the multiplication array to represent it.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <p>Combine concrete equipment into part whole models and bar models. Explore which are the parts and which is the whole. Match calculations to these. Give missing number calculations and solve practically. E.g. <math>\_\_ \times 8 = 24</math>.</p> </div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;">  </div> <p>Place fours (number pieces/arrangements of spots etc) on a counting stick or number line. Look at the fours and use to predict <math>\times 8</math>. Find <math>3 \times 4</math>, what will <math>3 \times 8 = ?</math></p> <div style="display: flex; align-items: center; justify-content: center; margin-bottom: 10px;">  </div> <p>Use counting stick methods (see page 7) to explore relationships between multiples of 8 and find more efficient ways to derive them using underlying skills and known facts. Ensure <math>11 \times 8</math> and <math>12 \times 8</math> is included. If they know <math>10 \times 8</math>, what would <math>20 \times 8 = ?</math> Make sure the relationship between <math>\times 9</math> and <math>\times 8</math> and the tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. Could you use it for <math>9 \times 8</math>? (E.g. <math>80 - 8</math>) <math>8 \times 9</math>? (<math>90 - 16</math>).</p>	<p>Solve problems and represent multiplication and division calculations by drawing arrays, number lines, part whole and bar models. Write calculations or problems to match visuals.</p> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;">   </div> <p>Use number relationships to fill in missing multiples in a number line. Explain how you found them.</p> <div style="text-align: center; margin-top: 10px;">  </div> <p>Use number lines and counting sticks to explore how many different ways you can find tables facts. E.g. <math>8 \times 8</math> could be <math>(10 \times 8) - 16</math>, double <math>4 \times 8</math> etc.</p> <p>Use bar models/part whole models/number lines to find missing numbers in multiplication and division. E.g. <math>\_\_ \times 8 = 32</math>  <math>48 \div \_\_ = 8</math>.</p> <p>Always, sometimes, never?      A multiple of 4, is also a multiple of 8.      A multiple of 8 is a multiple of 4.</p>

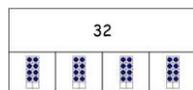
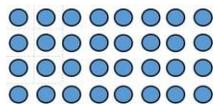
## 8x Table: Investigating Relationships

These strategies rely heavily on the ability to double so this must be secure.

See page 21 for introducing arrays.

It is important to show a range of strategies for  $\times 8$  so children begin to see the connections that exist within multiplication and have a number of strategies at their fingertips to access and apply facts quickly. Discuss the idea of making an amount 8 times greater. What would this look like on a number line? A bar model etc?

Match repeated addition and multiplication calculations to concrete and visual arrays.



Link 4x table to 8x. Make me an array to show  $4 \times 8$ . What do you think  $8 \times 8$  would be? Why? Make an array to check.

How can this bar model help you to find  $8 \times 8$ ?

Look at equipment such as Cuisenaire rods, cubes etc. How many eights? How many fours? Write multiplication facts to match. What would the division fact be?

True or false:  $3 \times 8 = 6 \times 4$ ? Show me how.

How many fours do you think would be in 9 eights?

Play a game. Show eights practically and ask children to predict on whiteboards the equivalent number of fours.

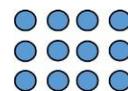
How many ways can you find  $8 \times 9$  (Examples may be;

Count in 8s, find  $4 \times 9$  and double it, find  $10 \times 8$  and take 8 off, find  $10 \times 9$  and take double 8 off). Examples could be given for children to prove with practical equipment.

Make an array to show  $8 \times 8$ . True or false: it also shows 2 lots of  $4 \times 8$ ? What about 4 lots of lots of  $2 \times 2$ ? What about  $4 \times 8 + 4 \times 8$ ?

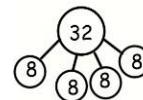
Can you explore to find other ways to break the array down into smaller equal groups? If there are 4 lots of 8, how many lots of 16 will there be?

Look at this array. What multiplication and division facts from the 4x table does it show? Can you use it to draw the fact for the 8x table?



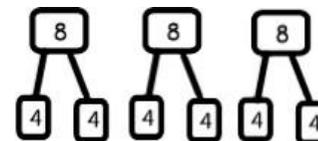
Colour in multiples of 4 and 8 in two different colours on a hundred square. What do you notice?

Fill in 4x table on multiplication grid. How could this help you find  $\times 8$  facts?



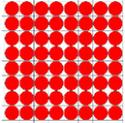
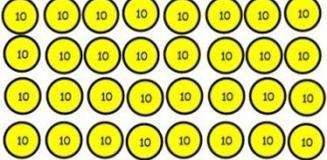
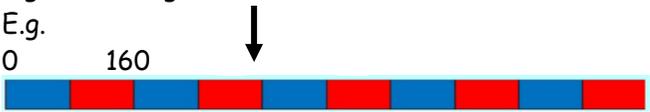
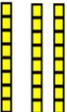
How could these help you find  $8 \times 8$ ?

$3 \times 8 = \_ \times 4$ ? Prove it (bar model etc).



Which of these facts could this visual show:  $6 \times 4$ ;  $3 \times 8$ ;  $(2 \times 4) + (4 \times 4)$ ;  $3 \times 2 \times 4$ ? Show visuals and explore the different calculations one visual could represent.

Draw something to show your strategy to find  $8 \times 9$ . Now draw a different way.

<p>Using visuals to break numbers down can help explore the distributive and associative properties of multiplication in an informal way.</p> <p>Understanding of place value can be developed by using concrete apparatus to link the strategies used with single digit numbers to larger numbers.</p>	 <p>Concrete materials and visuals can be used to develop understanding of multiplication with larger numbers, by making the links to single digit numbers explicit. Make an array with ones. Now make the same array with tens (or imagine the ones are tens).</p>  <p> <math>8 \times 40 = 320</math>    <math>40 \times 8 = 320</math>  <math>320 \div 8 = 40</math>    <math>320 \div 40 = 8</math>  OR  <math>80 \times 4 = 320</math>    <math>320 \div 4 = 80</math>  <math>4 \times 80 = 320</math>    <math>320 \div 80 = 4</math> </p> <p>A counting stick can also be used to make links between single digits and larger numbers.</p> <p>E.g.</p> 	<p>Draw the arrays made and circle the different groups found. Begin to explore how it might be represented in different ways. Could we show it on a bar model?</p> <p>If children are not secure with place value, drawing arrays where each circle/cross represents a ten instead of a one, can help consolidate their understanding.</p> <p>When showing the division link with larger numbers and practical equipment, sometimes it is appropriate to show it as sharing (e.g. <math>320 \div 4 = 80</math>) and sometimes as grouping (<math>320 \div 80 = 4</math>).</p>
<p>Use knowledge of doubles and halves to link quarters and eighths.</p>	 <p>Use a counting stick or number equipment showing eights. How many eighths are in 1 whole? 2? 3? etc. What do you notice? Find Cuisenaire rods to represent eighths and quarters. Investigate equivalence relationships between quarters and eighths and write calculations. E.g. 2 quarters = ___ eighths.</p>	<p><u>Link to fractions:</u></p> <p>Draw bar models and other pictures divided into eighths to find out many eighths are in whole numbers.</p> <p>Link quarters and eighths. How many quarters would 6 eighths make? 4 eighths make? etc. What do you notice? Show it on bar model/fraction wall.</p>

Reasoning to address misconceptions or help make generalisations.

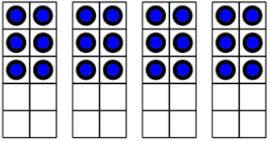
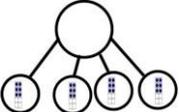
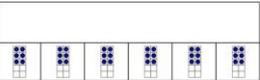
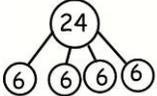
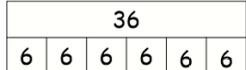
- Which is the odd one out:  $4+4+4+4+4+4+4$      $8+8+8+8$      $3+3+3+3$  Why?
- What's the same? What's different?     $8+8+8$      $3+3+3+3+3+3+3+3$
- True or false:  $5 \times 4 < 4 \times 8$ . Explain.

$2 \times 4 = 8$ .     $3 \times 4 = 12$ .     $4 \times 4 = 16$ .    What do you notice? What will  $4 \times 8 = ?$   
 $2 \times 8 = 16$ .     $3 \times 8 = 24$ .

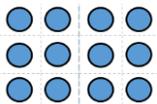
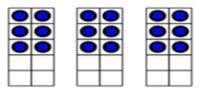
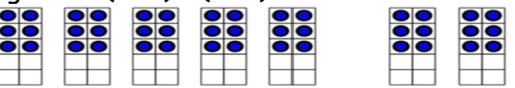
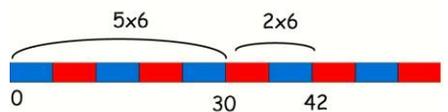
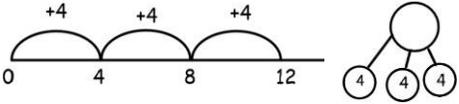
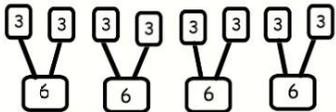
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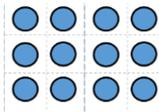
Louise says this represents 5 lots of 8. Is she right? What multiplication facts does it show? What division facts? Can you write a problem that the bar model might represent?

## 6x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Ensure doubling, halving and bridging strategies are secure.</p> <p>Make explicit that most of <math>\times 6</math> facts are already known from previous tables and revise these using the range of strategies already known. Only <math>6 \times 6</math> and <math>6 \times 7</math> are not known from another table or strategy.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Represent and solve problems using concrete apparatus initially to show relationships.</p>  $6 + 6 + 6 + 6 =$ $4 \times 6 = 24$ $24 \div 6 = 4$ <p>Use counting stick methods (see page 7) to explore relationships between multiples of 6 and find efficient ways to derive them using underlying skills and knowfacts. Ensure the link to <math>9 \times</math>, <math>11 \times</math> and <math>12 \times</math> is made through use of the tenth multiple.</p> <p>Make part whole models or bar models to represent calculations and problems. How many parts are needed? How many should there be in each part? What is the whole? Also, include missing numbers in different combinations.</p>   <p>Use to explore relationships. E.g. If this shows <math>6 \times 6</math>, what do you think <math>12 \times 6</math> will be?</p> <p><u>Fractions</u> Use Cuisenaire rods or other practical equipment. If each piece represents one whole, how many sixths are in 1? 2? 3? etc. What do you notice? Why does this happen?</p> <p><u>link:</u></p> <p>Write down all the fractions equivalent to one sixth. What do you notice? Mark a counting stick/number line in ones. How many sixths in 1? 2? 3? etc. Do you spot a pattern?</p>	<p>Colour the multiples of 6 on 100 square. Circle multiples of 3. What do you notice?</p> <p>Give out 2 digit numbers below 60. Which are definitely multiples of 6? Which can't be multiples of 6. How do you know? (E.g. odd numbers). Are there any facts that could help you quickly find out/check (e.g. link to multiples of other tables, such as threes or twelves).</p> <p>Represent multiplication and division calculations visually in different ways and use them to solve problems and find missing numbers. E.g. <math>6 \times \_ = 24</math>; <math>36 \div \_ = 6</math>.</p>   <p>Write multiplication and division calculations represented by each visual.</p> <p>Explore multiples rules through problems such as: <math>60 \div \_ = \_</math> Encourage children to work systematically. What definitely could go there? What definitely couldn't?</p>
<p>Reason to address misconceptions/generalise.</p>	<p>True or False: <math>9 \times 6 &gt; 6 \times 4</math>. Explain how you know.</p> <p><math>5 \times 6 &lt; \_</math> What could go there? What couldn't? Why.</p> <p>Sometimes, always never? Multiples of 3 are also multiples of 6.</p> <p>Show 100 square with some multiples of 5 and 6 circled. Spot which shouldn't be circled/are not multiples of 6.</p>	

## 6x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract						
<p>Make links between <math>\times 6</math> and <math>\times 3</math> and also between <math>\times 6</math> and <math>\times 5</math>.</p> <p>See page 21 for introducing arrays. Include problems which involve making an amount <math>6\times</math> greater and discuss how to represent.</p> <p>Explore distributive property of multiplication.</p>	<p><b>Concrete</b></p> <p>Make an array to show <math>3\times 4</math>. Write/match multiplication and division calculations to different arrays made.</p>  <p>Does the array show <math>3\times 4=12</math> or <math>4\times 3=12</math>? Explain.</p> <p>Investigate the relationships between 3 times table and 6 times tables. Show a number tripled on a counting stick and use it to predict the <math>6\times</math> fact.</p>  <p>Make me an array to show <math>3\times 4</math>. What do you think <math>6\times 4</math> would be? Why? Prove it.</p>  <p>Use a range of equipment showing sixes (e.g. dice, Cuisenaire rods etc) to show <math>3\times 3</math>. What would <math>3\times 6</math> be? Try with different amounts of threes. What do you notice?</p> <p>Now show sixes. So how many threes?</p>  <p>Use sixes to write facts from <math>6\times</math> table. Work out what the <math>3\times</math> facts would be. What do you notice?</p> <p>Use practical equipment, arrays and the counting stick to investigate the distributive property. E.g. <math>7\times 6 = (5\times 6) + (2\times 6)</math>.</p>  <p>What else could this show? E.g. <math>(2\times 6)+(3\times 6)+(2\times 6)</math>.</p> 	<p><b>Visual to Support Abstract</b></p> <p>Draw arrays (circles/crosses or rectangles drawn on squared paper) to show the facts from <math>6\times</math> table. Write 2 multiplication and 2 division calculations to match each array. (See page Inverse Relationships, page 12-15).</p> <p>Look at pictures of different visuals showing <math>3\times</math> table. Use them to find the related fact from <math>6\times</math> table. E.g. <math>3\times 4=12</math>, so <math>6\times 4=24</math>.</p>  <table border="1" data-bbox="1388 726 1601 805"> <tr><td colspan="3">12</td></tr> <tr><td>4</td><td>4</td><td>4</td></tr> </table> <p>xxxx xxxx xxxx</p> <p>Use visuals to find missing number facts (multiplication and division).</p> <p>True or false: <math>8\times 3=4\times 6</math>. Explain why. Draw something to prove it.</p>  <p>Explore different calculations that visuals can show. E.g. Above could show: <math>(3\times 3)+(3\times 3) + (2\times 3)</math> or <math>(2\times 3) + (6\times 3)</math> or <math>(2\times 6)+(2\times 6)</math>. Draw visuals to show calculations shown in different ways, e.g. <math>5\times 6+3\times 6</math>. Write an equivalent calculation.</p>	12			4	4	4
12								
4	4	4						

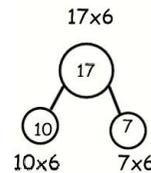


If this is  $4 \times 3$  what would  $4 \times 6$  be?  
 What does this array show? What if each counter was swapped for a tens counter? What could the array show now?  
 If  $3 \times 40 = 120$ , what does  $6 \times 40 = ?$   
 Write all the multiplication and division facts if the counters are worth one. Repeat, imagining the counters are tens. What if each counter represented 100? 10p?

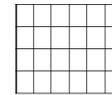
The distributive property should be explored as described when working practically and used in the context of real life problems.

The distributive property can be used when working with 2 digit numbers and can support the understanding of formal written methods. Concrete apparatus can be used in part-whole models and bar models, moving onto representing visually.

True or false:  $17 \times 6 = 10 \times 6 + 10 \times 7$ ?

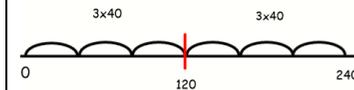


Draw an array on squared paper to show  $4 \times 6$ .



Imagine each square is worth 10, what facts could the array show now?

Visuals can be used to demonstrate and explain the distributive property and make links between known facts and larger numbers.

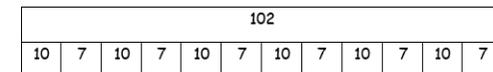
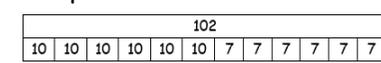
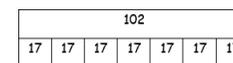


Explore the different calculations that the bar model below could represent. E.g.  $2 \times 6 + 2 \times 6$  or  $1 \times 6 + 3 \times 6$  etc.



Draw bar models, part whole models, number lines and arrays to show a range of calculations and problems. E.g. Jen had 3 boxes of 6 sweets and her friend gave her another 2 boxes of 6 sweets. How many sweets did she have altogether?  $3 \times 6 + 2 \times 6$ .

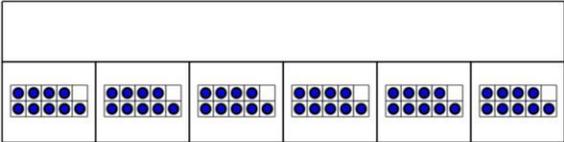
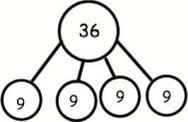
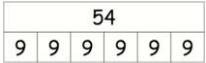
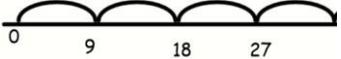
Which bar models could represent  $17 \times 6$ :



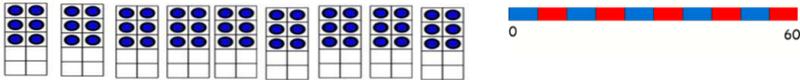
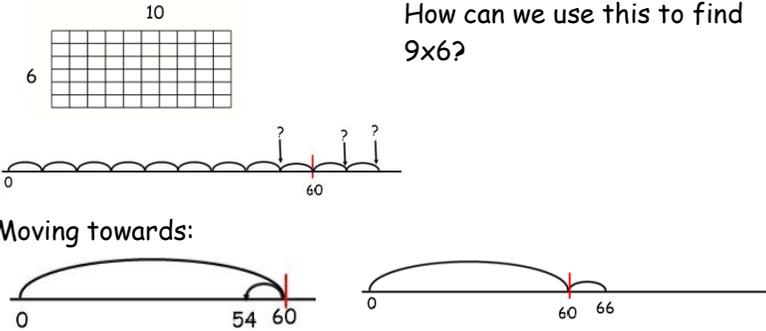
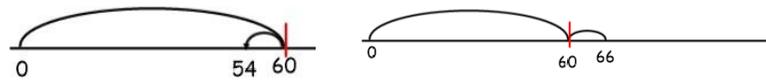
Reason to address misconceptions or help make generalisations.

- True or false: I can find the product of 5 and 4 by getting 4 lots of 5?
- Show 6 lots of 4 on a number line/bar model/part whole model. Now show 4 lots of 6. What's the same? What's different?
- Sophie says, " $2 \times 3 \times 4$  must be greater than  $4 \times 3 \times 2$ . Is she right? Explain how you know. Draw something to prove it.

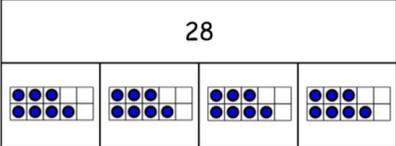
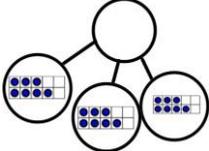
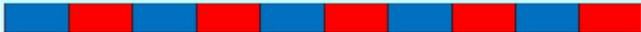
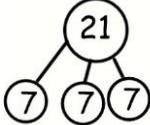
## 9x,11x,12x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Make explicit that most of <math>\times 9</math>, <math>\times 11</math> and <math>\times 12</math> facts are already known or can be quickly worked out using the tenth multiple. Practise using the 10<sup>th</sup> multiple before exploring skip counting.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Use counting stick methods (see page 7) to explore relationships between multiples of 9 and find efficient ways to derive them using underlying skills and known facts. (Repeat when working with 11x and 12x).</p> <p>Use a hundred square and circle any multiple of nine. Get a stick of ten cubes and add them. What do you notice? Predict where the next multiple will be? Is there an easy way to find it? (Add ten and subtract one). How could you show this on a number line? Would this strategy be useful for adding 8? How do you think it could help when adding 11 or 12?</p> <p>Use apparatus showing nines to represent and solve problems and missing number calculations using part whole models and bar models for each table (9x,11x,12x). Explore which are the parts (how many parts, how many in each part) and which is the whole in calculations and problems.</p>  <p>Investigate flexible strategies when adding nine, such as swapping the ones to make an easier calculation. E.g. <math>27+9=20+7+9</math>, so think of it as <math>29+7</math> then imagine the one moving over so it becomes <math>30+6</math>.</p>	<p>Record the multiples of nine when working with a hundred square. What do you notice? What is happening to the tens digit each time. What is happening to the ones digit? Why? Repeat for 11x and 12x.</p> <p>True or false: if a number is a multiple of 9, its digits will always equal 9? Does this work with any multiple of 9? What about multiples of other numbers?</p> <p>Write repeated addition, multiplication and division calculations to match arrays, number lines, part whole models and bar models. Draw visuals to represent and solve calculations and missing number problems. E.g. <math>36 \div \underline{\quad} = 9</math></p>    <p>True or false: The part whole model shows <math>2 \times 9 + 2 \times 9</math>. What else could it show?</p>

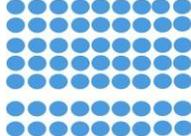
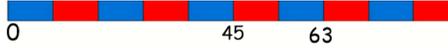
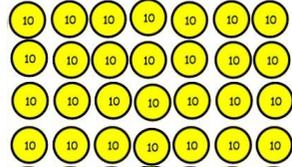
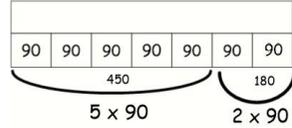
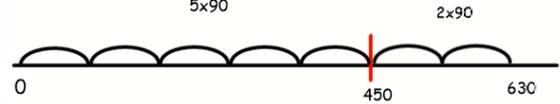
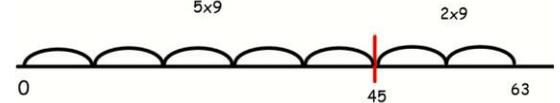
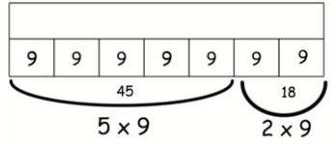
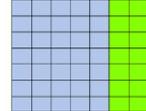
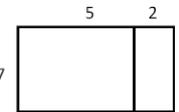
## 9x, 11x, 12x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract
<p>It is important to see the link between <math>\times 10</math> and <math>\times 9</math> (or <math>11x</math>, <math>12x</math>) and make use of the commutative property as this is generally the most efficient method. Also, investigate other relationships to give a range of methods to access different facts.</p>	<p>Revise the facts from <math>9x</math> that are already known. Point to the tenth multiple on counting stick, number line and use practical apparatus. Ask children to work out <math>9x</math> for all tables covered so far. (Repeat with <math>11x, 12x</math>).</p> <p>What tables fact are shown below? How can they help us find <math>9 \times 6</math>?</p>  <p>E.g. If this is 60, what multiple are we counting in? What would <math>9 \times 6</math> be? How do you know? How about <math>11x, 12x</math>? How about if the tenth multiple is 50? What would <math>9x</math> be? etc. (Repeat ideas when working with <math>11x</math> table and <math>12x</math> table). If <math>10 \times 9 = 90</math>, what would <math>20 \times 9 = ?</math> Could this help you find <math>19x</math>?</p> <p>Investigate the relationship between the threes and nines using equipment such as Cuisenaire rods, Numicon etc</p>  <p>If this is <math>3 \times 3</math>, what would <math>6 \times 3</math> look like? What about <math>9 \times 3</math>? Explain what you notice?</p> <p>Match repeated addition, multiplication and division calculations to the apparatus. Use apparatus to represent different calculations and to solve problems.</p> <p>Investigate the relationship between the sixes and twelves. Put out apparatus for <math>6x</math> table. Show arrays or number line visuals from <math>3x</math> or <math>6x</math> table. Use them to find facts from <math>12x</math> table.</p>	<p>Draw number lines and arrays to show <math>10x</math> fact from different tables and use it to find <math>9x</math>. (Repeat when working with <math>11x, 12x</math>).</p>  <p>How can we use this to find <math>9 \times 6</math>?</p> <p>Moving towards:</p>  <p>Draw rectangles to show arrays from the <math>9x</math> table. Give out cards with 2-digit numbers on. Which will be multiples of 9? Draw rectangles on squared paper to make arrays and find out. What do you notice about all the multiples of 9? Fill out <math>3x</math>, <math>6x</math> and <math>9x</math> facts on a multiplication grid. What do you notice? Fill out the <math>6x</math> and <math>12x</math> times facts. What do you notice? What would happen if we doubled the <math>12x</math> fact? Look at a calculation. What could it mean? How many ways can you find to solve it? Draw the two ways you find easiest. E.g. <math>12 \times 4</math></p> 

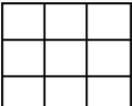
## 7x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
<p>Compare repeated addition and multiplication calculations.</p> <p>Ensure examples which include multiplying by 1 and 0 are included.</p>	<p>Use concrete apparatus in sevens to represent and solve problems. Match repeated addition, multiplication and division calculations to each other and to concrete representations. In what other ways could repeated additions be written? E.g. <math>7+7+7+7+7 + 7+7</math> could be written as <math>5 \times 7 + 2 \times 7</math>. Match calculations to bar models and part whole models. Discuss which are the parts and which is the whole and use this help with solving missing number calculations and problems.</p> <p>E.g. <math>\underline{\quad} \times 7 = 28</math>. <span style="margin-left: 100px;"><math>\div 7 = 3</math>.</span></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="text-align: center; margin-top: 10px;">  </div> <p>Use counting stick methods (see page 7) to explore relationships between multiples of 7 and to find efficient ways to derive them using underlying skills and known facts.</p> <p><u>Fractions link:</u></p> <div style="text-align: center; margin-bottom: 10px;">  </div> <p>Use Cuisenaire rods which represent the seven (or other concrete equipment). How many sevenths in 1 piece? 2 pieces, 3 pieces etc.</p> <p>Mark a counting stick/number line in ones. How many sevenths in 1? 2? 3? etc. Do you spot a pattern?</p>	<p>Use representations to explore relationships and solve problems.</p> <div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: left;"> <p>The part whole model shows <math>3 \times 7</math>. Use it to help you find <math>6 \times 7</math>.</p> <p>Use the bar model to help you find <math>8 \times 7</math>.</p> </div> </div> <div style="text-align: center; margin-top: 10px;">  </div> <p>Draw a bar model or number line to show <math>3 \times 7 + 2 \times 7</math>.</p> <p>Use a number line to show the relationships from the counting stick and find missing numbers.</p> <div style="text-align: center; margin-top: 10px;">  </div> <p>Use a bar model or a number line to find the answer: <math>\underline{\quad} \times 7 = 63</math>.</p> <p>Write a problem that the calculation could represent.</p> <p><u>Fractions link:</u> Write down all the fractions equivalent to one seventh. What do notice? Sort fractions that are equivalent to one seventh and those that are not.</p>

## 7x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract
<p>Investigate commutative property. Find many different ways to solve same calculations.</p> <p>See page 21 for introducing arrays.</p> <p>Explore distributive property, as this can be helpful for children who can't use other methods to recall the x7 facts, e.g. use of <math>(x5) + (x2)</math> for x7.</p>	<p>Use practical equipment to highlight that most of the x7 facts are already known from other tables. Write multiplication and division calculations to match arrays. Continue to use a range of representations to illustrate the distributive property and show how x5 and x2 facts can be used to find x7.</p> <p>E.g. <math>7 \times 9 = 5 \times 9</math> and <math>2 \times 9</math>.</p>   <p>Concrete and visuals can be used in the same way to help with the application of tables facts with larger numbers.</p>  <p><math>4 \times 70 = 280</math>    <math>280 \div 70 = 4</math>  <math>70 \times 4 = 280</math>    <math>280 \div 4 = 70</math>.</p> <p>True or false: This array could also show <math>40 \times 7</math>?    <math>280 \div 7</math>?</p>   <p>How can <math>6 \times 7</math> help you find <math>60 \times 7</math>? Make/draw an array with tens counters to show me. I know <math>6 \times 7 = 42</math>. What else do I know?</p>	<p>Represent visually on number lines, bar models etc.</p>   <p>Draw rectangles to show arrays which represent facts in the 7x table. Show how this can split into the x5 fact and the double.</p>  <p>E.g. <math>7 \times 7 = 5 \times 7</math> and <math>2 \times 7 = 35 + 14 = 49</math>.</p> <p>As children become confident, blank arrays can also be drawn to save time.</p>  <p>E.g.</p> <p>True or false: <math>6 \times 7 &gt; 7 \times 5</math>. Explain your thinking.</p> <p>How many ways can you find to solve this tables fact: <math>7 \times 8</math>. What known facts and strategies could help you? E.g. Find <math>7 \times 4</math> twice, find <math>5 \times 8 + 2 \times 8</math>, find <math>10 \times 7</math> an subtract two sevens. Visually represent the different ways of thinking about the calculation.</p>

Exploring Tables Facts through Common Multiples and Factors, Prime, Square and Cube Numbers.

Curriculum Links	Notes	Concrete	Visual to Support Abstract
<p>Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes (Y5)</p> <p>Recognise that shapes with the same areas can have different</p>	<p>Encourage children to predict and generalise as they are working with practical apparatus. Also, make links to larger numbers and decimals, What if each</p>	 <p>Use multi-link cubes to make as many different rectangles as you can out of a given numbers of cubes. Find factor pairs.</p> <p>Predict all of the factors for each number given. Write calculations to match. Predict based on doubles/halves relationships. E.g. If 10 is a factor, what else will be a factor?</p> <p>Are there any numbers that will only make one rectangle? Predict which numbers will only make a square (no other rectangles). Why are these called square numbers?</p>	 <p>Draw rectangles to represent practical work. Predict and generalise about relationships within multiplication. E.g. If a 3x8 rectangle was drawn, if you double one side, what happens to the other side? If <math>60 \div 6 = 10</math>, what would <math>60 \div 3</math> be? What about <math>60 \div 12</math>? Etc.</p> <p>What do you notice about the numbers that will only form a square? Can you find any others?</p>

perimeters and vice versa (Y6)  
 Establish whether a number up to 100 is prime and recall prime numbers up to 19. (Y5)  
 Recognise and use square numbers and the notation for squared ( $^2$ ) cube numbers and the notation for cubed ( $^3$ ) (Y5).  
 Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres ( $\text{cm}^3$ ) and cubic metres ( $\text{m}^3$ ), and extending to other units (e.g. mm and km). (Y6)

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers (Y5)  
 Identify common factors, common multiples, prime numbers (Y6).

cube was worth 10, 100, 0.1 ?etc.

Use multi-link cubes or draw rectangles to systematically investigate all factors of a number. Record on factor rainbow.

Find out which numbers from 1-20 can only make one rectangle made from a single line of cubes. E.g.



Consider the numbers up to 20. Which cannot be prime? Explain why.

Get 16 multi-link cubes. How many different cuboids can you make? Write the multiplication sentences to go with each. Try with different numbers of cubes. Investigate which numbers can build a cube? Why is this?



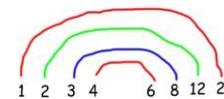
Match/write different calculations to go with each cube/cuboid? Make cuboids of given dimensions e.g.  $6 \times 3 \times 2$ .

Show a cuboid and give its dimensions. Can the children quickly calculate its volume? Discuss the different ways it was solved.

Make one face of a cube. Give to a partner. What cube number would it make? Build it to check. Explore the relationship between square and cube

A rectangle has an area of 48. What could its sides be? Represent as a factor rainbow.

Factor rainbow for 24



What could its perimeter be?



Draw rectangles to identify which numbers only have one possible rectangle (where the cubes make one line).

Sort numbers up to 20 according to whether or not they are prime. Which numbers do you know definitely can't be prime?

Give out square numbers and prime numbers on cards and draw rectangles to represent them. Predict which will be prime? Which will be square? Draw to prove it.

Draw cuboids and write/match calculations to go with them.

Write equivalent calculations

$$\_\_ \times \_\_ \times \_\_ = \_\_ \times \_\_ \times \_\_$$

Predict a calculation for another cube with the same volume.

$3 \times 4 \times 5 = \_\_ \times 2 \times 6$ . Use cubes to check. Discuss different ways to solve.

Use this to revise use of factors to find more efficient ways to multiply (i.e. associative law). E.g.  $7 \times 16 = 7 \times 8 \times 2 = 56 \times 2$  or  $112$ .

Fractions link:  
Compare and order fractions whose denominators are all multiples of the same number. (Y5)  
Fractions link:  
Use common factors to simplify fractions.  
Use common multiples to express fractions in the same denomination (Y6)  
Add and subtract fractions with the same denominator and denominators that are multiples of the same number (Y6)

Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and  $\times 3 = \text{division facts}$  (Y6).

Finding common multiples involves doubling, tripling and bridging strategies from previous years.  
Look back to strategies if children not secure.

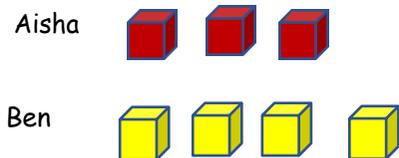
Problems such as these can provide opportunities for halving (by using halving and halving again to divide by 4) and tripling strategies.

numbers. Show cubes for children to predict the volume.  
Use a counting stick to find common multiples using two different coloured sticky notes for the different times tables.  
E.g. Find the common multiples of 24 and 60.



Jessica thinks 32 is a common multiple of 8, 12 and 16. Is she right? Explain how you know.

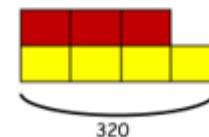
Use Cuisenaire rods or cubes with bar models to show ratio.  
E.g. For every £3 Aisha saves, Ben saves £4. Ben has saved up £320. How much has Aisha saved?



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

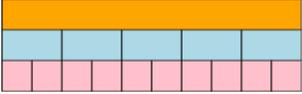
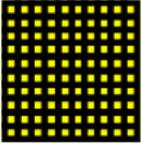
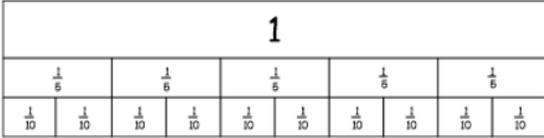
Use different coloured pens to highlight the individual times tables and then identify which ones they have in common to find common factors/multiples.

Draw bar models to show/work out relationships.



= 320  
 $320 \div 4 = 80$   
 = 80  
 $\pounds 80 \times 3 = \pounds 240$

Consolidation through Fractions, including Decimals and Percentages).

Curriculum Links	Notes	Concrete	Visual to Support Abstract
<p>Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths. (Y5)</p> <p>Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions. (Y6)</p> <p>Compare and order fractions, including fractions <math>&gt;1</math>. (Y6)</p> <p>Divide proper fractions by whole numbers (e.g. <math>1/3 \div 2 = 1/6</math>). (Y6)</p>	<p>Make links between thirds and sixths, quarters and eighths, sixths and twelfths etc by using doubling and halving strategies.</p>	<p>Revise underlying relationships (e.g. doubles/halves) for each table when working with fractions.</p> <p>For example, when working with fifths, arrange Cuisenaire rods into a bar model then discuss how many tenths it is equivalent to. Discuss the link between <math>\times 5</math> and <math>\times 10</math> in whole numbers and tenths and fifths in fractions.</p>  <p>How many tenths? How many fifths? What do you notice? How many fifths in 2? How many tenths in 2? Use a counting stick as one whole. Mark on fifths, tenths etc and discuss relationships. If this is one whole, what would a tenth look like? What would a fifth look like?</p>  <p>Put Dienes or place value counters in bar models and compare relationships between fifths and tenths. Repeat to link quarters and eighths, thirds and sixths etc.</p>	 <p>Karen says that you can find one tenth by doubling one fifth? Is she right? Prove it.</p> <p>Colour one tenth on a 100 square if the 100 square represents one whole. Colour one fifth etc. Record what you colour as a decimal, as a percentage, as tenths/hundredths etc.</p> <p>Revise strategies for individual tables when working with fractions. E.g. If <math>3/8=21</math>, what will <math>6/8</math> be?</p> <p>Write down all the fractions equivalent to <math>1/3</math>. What do you notice? Repeat with other fractions.</p> <p>Which of the fractions equivalent to one third can you make equivalent to one sixth? Show it on a bar model. Predict those which you won't be able to convert to one sixth. Explain why.</p>

<p>Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements as a mixed number (E.g.: <math>\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}</math>) (Y6)</p>		<p>How many eighths are in 4 whole ones? How can you find out? Use equipment such as Cuisenaire rods, Numicon or number lines/counting sticks to explore. Look to appropriate year group to reinforce tables strategy when working with given fractions.</p> <p>How many eighths in 2 whole ones? How can this help you work out how many in 4 whole ones?</p>	<p>Reinforce tables facts through work with mixed numbers.</p> <p>E.g. True or false: <math>\frac{15}{4} &gt; \frac{23}{8}</math>. Explain how you know.</p>
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# Whole School Strategies:

- The Counting Stick
- Multiplication Grids
- Multiplication Wheels



## The Counting Stick

This is a wooden stick, one metre long, divided into ten equal parts. If you don't have one, a metre stick can easily be converted using tape to divide. An empty number line can also be used in the same way. This is a powerful way to represent number relationships and can also be used in Foundation Stage for counting in ones and finding missing numbers.

**The example given is for 3x table but the same strategy applies regardless of the table being taught.**

When first using with younger children, use sticky notes with the multiples already written on, gradually removing each multiple in the order given and explaining how we can work out the missing multiple even when it isn't there using the other multiples. After removing each multiple, count through the multiples again, forwards and backwards, including those which have been removed. As children become more confident, you could begin with empty counting stick and use the 10<sup>th</sup> multiple to work out the first.

What is the first multiple? What table are we exploring?



Double the first multiple to find 2x



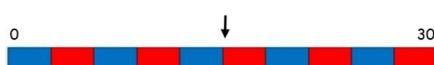
Discuss how the 4<sup>th</sup> multiple can be found by doubling the 2<sup>nd</sup> multiple.



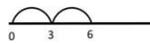
The 9<sup>th</sup> multiple can be found by subtracting one multiple from the 10<sup>th</sup> multiple, i.e. 30-3).



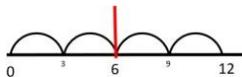
5<sup>th</sup> multiple - halve the tenth multiple).



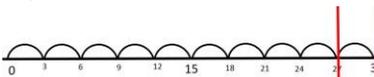
2x3



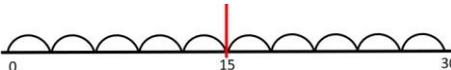
4x3



9x3



5x3



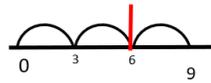
3<sup>rd</sup> multiple - triple the first multiple or double then add multiple.



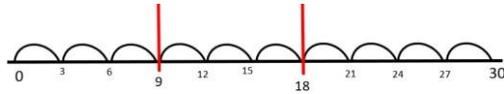
6<sup>th</sup> multiple - double the third multiple,



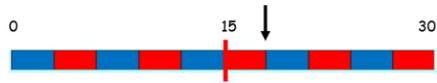
3x3



6x3



Or find 5x and add another multiple.

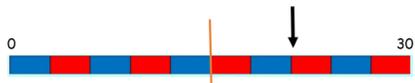


8<sup>th</sup> multiple - double the 4<sup>th</sup> multiple,

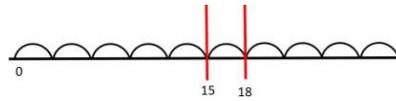


or remove the double from the 10<sup>th</sup> multiple.

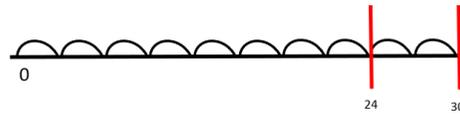
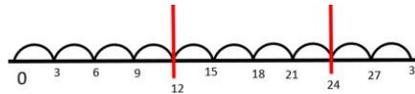
7<sup>th</sup> multiple - find 5x and add the double (2x).



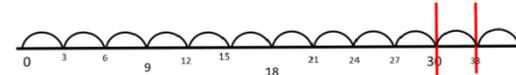
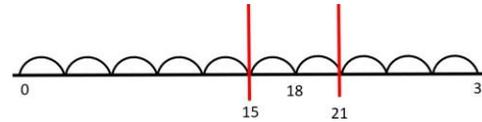
Discuss how 11x and 12x could be found.



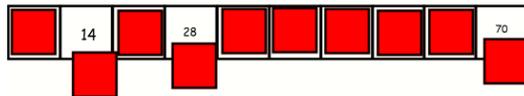
8x3



7x3



- Younger children may make their own number line by putting sticks of cubes of appropriate multiples along it and marking what number it would be to gain initial understanding of counting stick.
- Number cards with multiples on can be put face down onto an empty number track and a game played where an adult has to point to a card and the child predicts what it will be, explaining how they know. Begin with the easiest multiples first (e.g. first, second, ninth etc). The adult can use their turn to model language and the strategy used. E.g. I knew it (4<sup>th</sup> multiple) would be 28, because I knew 4x would be double 2x. This can be made as a homework game or played with the class on a number line.



- Muddle up the multiples, or swap around two multiples, and ask the children to put them in the correct order, explaining why it was wrong in the first instance.
- Use the counting stick without any multiples on it, or only one, so the children have to use their knowledge of doubling, tripling, halving to derive the multiples of a given number. This can help with finding the multiples of larger 2 digit numbers for long division.
- Use alongside practical equipment (e.g. place value counters) to help develop understanding of place value with larger numbers and decimals. This relieves memory load by showing how the relationships stay the same, so the same strategies can be used.

E.g.



Or



The language used and questions asked can also show the link to division.

E.g.  $28 \div 7 = ?$  Or how many sevens are there in 28?

I know there are 2 sevens in 14, so there must be 4 sevens in 28.



For a video example see: Times Tables in Ten Minutes. Available at:

<https://www.youtube.com/watch?v=yXdHGBfoqfw>.

Please note, this is not to show a method to memorise the 17x table, but a strategy to show how to use number relationships to find multiples of any of the times tables.

## Multiplication Grids

With older children, multiplication grids can be rearranged to highlight the links between different tables, investigate patterns and help reduce memory load. Some examples are given below.

<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>1x</th> <th>2x</th> <th>3x</th> </tr> </thead> <tbody> <tr><td>1</td><td>1</td><td>2</td><td></td></tr> <tr><td>2</td><td>2</td><td>4</td><td></td></tr> <tr><td>3</td><td>3</td><td>6</td><td></td></tr> <tr><td>4</td><td>4</td><td>8</td><td></td></tr> <tr><td>5</td><td>5</td><td>10</td><td></td></tr> <tr><td>6</td><td>6</td><td>12</td><td></td></tr> <tr><td>7</td><td>7</td><td>14</td><td></td></tr> <tr><td>8</td><td>8</td><td>16</td><td></td></tr> <tr><td>9</td><td>9</td><td>18</td><td></td></tr> <tr><td>10</td><td>10</td><td>20</td><td></td></tr> </tbody> </table>		1x	2x	3x	1	1	2		2	2	4		3	3	6		4	4	8		5	5	10		6	6	12		7	7	14		8	8	16		9	9	18		10	10	20		<p>Fill in the tables grid for your 1 and 2 times table. Now fill it in for your 3x table. What do you notice? This strategy could be adapted to show the link between 10x, 5x and 15x.</p>	<p>Use the tens times table to help find your 5x facts.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>10x</th> <th>5x</th> </tr> </thead> <tbody> <tr><td>1</td><td>10</td><td></td></tr> <tr><td>2</td><td>20</td><td></td></tr> <tr><td>3</td><td>30</td><td></td></tr> <tr><td>4</td><td>40</td><td></td></tr> <tr><td>5</td><td>50</td><td></td></tr> <tr><td>6</td><td>60</td><td></td></tr> <tr><td>7</td><td>70</td><td></td></tr> <tr><td>8</td><td>80</td><td></td></tr> <tr><td>9</td><td>90</td><td></td></tr> <tr><td>10</td><td>100</td><td></td></tr> </tbody> </table> <p>This could be adapted to explore multiplying by twenty or to explore the link between <math>\times 1</math> and <math>\times 0.5</math> (or <math>\times \frac{1}{2}</math>) as well as other obvious doubling relationships (twos/ fours/ eights or threes /sixes/ twelves).</p>		10x	5x	1	10		2	20		3	30		4	40		5	50		6	60		7	70		8	80		9	90		10	100	
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<p>Fill in the tables grid for your 10 and 1 times table. Now fill it in for your 9x table.</p>	<p>What do you notice? Why does this happen? Can you prove it? Does it work with other numbers? Can you use this to find new facts?</p>	<p>Fill in the tables grid for your 10 and 2 times table. Now fill it in for your 8x table.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th></th> <th>10x</th> <th>2x</th> <th>8x</th> </tr> </thead> <tbody> <tr><td>1</td><td>10</td><td>2</td><td></td></tr> <tr><td>2</td><td>20</td><td>4</td><td></td></tr> <tr><td>3</td><td>30</td><td>6</td><td></td></tr> <tr><td>4</td><td>40</td><td>8</td><td></td></tr> <tr><td>5</td><td>50</td><td>10</td><td></td></tr> <tr><td>6</td><td>60</td><td>12</td><td></td></tr> <tr><td>7</td><td>70</td><td>14</td><td></td></tr> <tr><td>8</td><td>80</td><td>16</td><td></td></tr> <tr><td>9</td><td>90</td><td>18</td><td></td></tr> <tr><td>10</td><td>100</td><td>20</td><td></td></tr> </tbody> </table>		10x	2x	8x	1	10	2		2	20	4		3	30	6		4	40	8		5	50	10		6	60	12		7	70	14		8	80	16		9	90	18		10	100	20																																		
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Dev says that if you know your doubles and your five times table, you can easily find your 7x facts. Is he right? Explore different numbers on a multiplication grid to find out. Explain what you find out.

Number to test	5x	Double (x2)	7x
3	15	6	21

It is important to focus upon making links to known facts and stressing that often one fact can help find another. Encourage children to identify those tables fact they find most difficult to remember and which are the easiest for them. Are there any ways they could use what they know already to help them find new facts?

Rather than the traditional multiplication grid which shows 100 facts (for up to 10x10), it can be less daunting to make use of the commutative property and take out the facts that are already known from other tables. This reduces the number of facts to learn from 100 to 55. When the x1 and x10 facts are removed, only 36 are left. 26 of these are in the twos, threes, four or five times table, so it is worth focusing on the doubling, halving and bridging skills required to find facts from these tables quickly. Providing a strong focus has been maintained on doubling, halving, children should be able to use these to quickly find the 7 facts still unknown from the six and eight times table. This leaves 9x7, 9x9 and 7x7 to learn and children should already have an efficient strategy for multiplying by 9. This only leaves 7x7. Breaking down tables facts in this way can help both the teacher and the children feel less overwhelmed!

1x1										
2x1	2x2									
3x1	3x2	3x3								
4x1	4x2	4x3	4x4							
5x1	5x2	5x3	5x4	5x5						
6x1	6x2	6x3	6x4	6x5	6x6					
7x1	7x2	7x3	7x4	7x5	7x6	7x7				
8x1	8x2	8x3	8x4	8x5	8x6	8x7	8x8			
9x1	9x2	9x3	9x4	9x5	9x6	9x7	9x8	9x9		
10x1	10x2	10x3	10x4	10x5	10x6	10x7	10x8	10x9	10x10	

A grid similar to the one above can be used as a starting point to discuss different strategies for finding tables facts. Eg. Point to  $9 \times 6$  and ask the children to explain and show different ways to find that fact. Examples may be: Doubling  $9 \times 3$ , finding  $10 \times 6$  and subtracting 6, finding  $9 \times 5$  and adding 9 etc. Strategies can be shown on number lines, with arrays (or blank rectangles drawn to represent arrays with larger numbers) or bar models to help clarify thinking and to make explicit how the relationships and properties that exist within multiplication can be used to relieve memory load and find new facts.

### The Multiplication Wheel

This is a great way of teaching how to use their known facts to calculate those not known. If a child knows one of the multiple they can double for 2, double that for 4, double that for 8. Then if they know two of the multiple and one of it they can sum it for three of the multiple and so on.

